SHOWING ADS TO THE WRONG CONSUMERS: STRATEGIC INEFFICIENCY IN ONLINE TARGETED PAY-PER-CLICK ADVERTISING

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Abstract

Online advertising is different from newspaper, television and other traditional advertising, because 1) Google, Amazon, Facebook and other online advertising platforms are better able to personalize ads to fit consumers characteristics and 2) each merchant pays per consumer that clicks on its ad. This paper analyzes online targeted pay-per-click advertising. Because online advertisement platforms maximize clicking and do not maximize the merchants profits, I find that a platform is induced to show ads inefficiently, showing an ad to some consumers who would not buy the product and not showing the same ad to other consumers who would.

1 Introduction

Whenever consumers search, message, buy and surf online, they are bombarded with pop-up ads, banner ads and sponsored-link ads. Online advertising has become an integral part of consumers' cyber-lives and in many cases an integral part of their real lives. Computing has displaced the radio as our second most time-consuming media outlet (Council for Reasearch Excellence, 2009).[1] At the same time, online advertising has become the third largest advertising market in the United States (PricewaterhouseCoopers, 2010). Yet, online advertising is different from traditional advertising through a much higher prevalence of

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[1]Television remains the most time consuming media outlet.

"targeted advertising" and a unique "pay-per-click" (PPC) method of charging for advertisements.

Targeted advertising is when different ads are shown to different consumers based on tastes, locations or demographics. By advertising for Geno's Cheesesteaks in The Philadelphia Inquirer or for Meow Mix in Cat Fancy, advertisers hope to increase the effectiveness of their ads. Yet targeted advertising is more prevalent and more precise on the internet, because Google, Amazon, Facebook and other online advertising platforms are better able to personalize ads to fit consumers characteristics through more information about consumers and more precise computing technology. Google tracks what content we view and what searches we perform through temporary internet files and uses it to show us personalized ads.[2] Amazon uses our shopping history to recommend products and services to us.[3] Facebook uses our profiles to personalize ads by age, gender, keywords, education, workplace, relationship status, relationship interests and languages.[4]

Most online advertising uses *pay-per-click* (PPC) pricing.[5] Here a merchant pays for his ad based on the number of consumers who click on his ad or visit his site instead of based on the number of consumers that see his ad or buy his product. This pricing system is practically nonexistent in the other forms of advertising like newsprint or television. We do not see car dealers paying for newspaper or television ads based on the number of people the ad inspires to visit their store. Instead we see car dealers paying based on the expected sales the ad will generate or the number of people who see the ad.

Some previous literature has examined targeted advertising (ex.: Johnson, 2009; Van Zandt, 2004), pay-per-click advertising (ex.: Agarwal et al., 2009) and the role of advertising platforms (ex.: Ghose and Yang, 2009). Yet only this paper and an extension of the basic model presented in Cornière (2009) analyze how an online advertising platform would target advertise in pay-per-click pricing. Cornière (2009) assumes that consumers are uniformly spatially distributed and that an ad platform can only "threshold targeted advertise." I define *threshold targeted advertising* as showing an ad to those consumers with reservation prices for the good above some threshold value and not showing the ad to all consumers with reservation prices for the good below the same threshold value. I relax these assumptions by allowing for a general demand function and allow the ad platform to choose more complex targeting strategies. I find that an ad platform would not choose to threshold targeted advertise.

To do this I adapt the classic costly search model. In these models consumers do not know how much they will value a product, their reservation price, until they travel to a store or click on an ad, wasting time and energy. The idea is that you do not know if you will like a sports car until you hear the roar of its engine and take it for a test drive. And you do not know if you will like that

[2]Refer to http://www.google.com/intl/en_us/ads/ads_1.html/ for details.

[3]Refer to http://www.amazon.com/gp/seller-account/mm-summary-page.html/ref=gw_m_b_awus?ie=UTF8&ld=AZAdvertiseMakeM&topic=200260730 for details.

[4]Refer to http://www.facebook.com/advertising/ for details

[5]This is sometimes called cost-per-click (CPC) pricing.

Gershwin album until you click on its ad and listen to a sample of Ella belting that sweet melody. Because consumers do not know their reservation prices, a consumer will only click on the ad or travel to the store, if their expected benefit from doing so is more than the search cost, which is the opportunity cost or travel cost.[6] You would not visit the sports car dealer unless there was some chance you would buy a car, nor would you click on an ad if you never buy anything online.

The advertising platform is maximizing its profit that it generates from the pay-per-click ads, not the profit of the firm. If the price of a click is sensitive to the firms profit in a way that makes the advertisement platform's profit a fixed proportion of the firm's profit, then the advertisement platform would be maximizing the profit of the firm. However, that would be a very unusual example of pay-per-click price sensitivity. In this paper I explore the simple case when pay-per-click price is fixed. This example of price sensitivity exists. For example, many online advertising platforms, including Google, provide a minimum pay-per-click price. Here, an advertising platform maximizes clicking on the ad, not the profit of the firm.[7] After examining this case, I relax this assumption by allowing the ad platform to make a take-it-or-leave-it offer to the firm when choosing how to target the ad.

When an advertising platform chooses who it shows an ad to, the advertising platform is choosing the demand curve for the product. When the platform is maximizing clicking, it will not show the ad to *some* people that it would rationally expect to buy the product in order to change the shape of the demand curve, inducing online merchants to lower their prices. This increases the expected benefit from clicking, leading to more clicking. In this way, an advertising platform would show an ad to some consumers who it rationally expects not to buy the product and not show the same ad to other consumers who it rationally expects to buy the product. Google would show an ad for Gershwin album to a punk teenager and not to a music professor. And Amazon would show an ad for a book on game theory to a garage mechanic and not show the same ad to me.

2 Literature Review

In this section, I explore several common assumptions made in the existing economics literature on targeted advertising. I explain how my model relaxes these assumptions.

Most of the economics literature on targeted advertising assumes that an advertiser chooses which consumers are shown its ad (ex.: Bergemann and Bonatti, 2010; Iyer et al., 2005; Anand and Shachar, 2009; Chandra, 2009). Yet in online advertising, profit-maximizing ad platforms like Google choose which consumers are shown each ad. Cornière (2009) was the first to explore how a

[6]This assumes there is no added utility from shopping.
[7]Refer to https://adwords.google.com/select/KeywordToolExternal for details.

profit-maximizing online ad platform would choose which consumers are shown an ad. My model extends this analysis to a more general framework.

My model relaxes two important important assumptions made in Cornière (2009): he assumes that consumers are uniformly spatially distributed and that an ad platform can only threshold targeted advertise. Cornière (2009) shows that an advertiser would choose to show its ad to those consumers above a threshold value. Cornière (2009) assumes that an ad platform would choose to show an ad in the same way. I relax these assumptions by allowing for a general demand function and allow the ad platform to choose more complex targeting strategies. I find that an ad platform would not choose to threshold targeted advertise. Instead, it will not show the ad to some people that it would rationally expect to buy the product in order to change the shape of the demand curve, inducing online merchants to lower their prices.

Bergemann and Bonatti (2010), Chandra (2009), Iyer et al. (2005) and Johnson (2009) assume a box-shaped demand curve instead of a downward sloping demand curve. In their models, each consumer i has a probability λ_i of having a reservation price of r for the product and a probability $1 - \lambda_i$ of having a reservation price of 0 for the product. The firm advertises to consumers with higher probabilities (the "λ_i"s) of being willing to buy the product for r. In reality, firms should advertise not only to consumers more likely to buy their products, but also advertise to consumers willing to pay more for their products. The selected composition of the audience of a firms targeted advertising campaign should influence its pricing decision. Cornière (2009), Esteban et al. (2001) and my model use downward sloping demand curves so that we can explore endogenous product prices.

3 The Model

I develop a costly search model with a two-sided market and find a sub-game perfect Nash Equilibrium. My model is similar to the Bertrand-Chamberlin-Diamond search models with a one-sided market found in Wolinsky (1986) and Anderson and Renault (1999), and most similar to Cornière (2009) and Anderson and Renault (2006). In Anderson and Renault (2006), a firm encourages some consumers to click on its ad or travel to its store through the content of its advertising. Their advertising content acts similarly to targeted advertising in my model, signaling a consumer whether or not to click on an ad. The key difference here is the addition of an advertising platform and pay-per-click pricing. Like Cornière (2009), I find that an advertising platform would not target-advertise in the same way that a firm would. Not only would the ad platform not set the same targeted advertising threshold (as shown in Cornière, 2009), but an advertising platform would not threshold targeted advertise.

This section is split into three subsections. The first subsection shows what motivates each of the three types of agents and what actions they can perform, while the second subsection introduces the general idea of the model. In the third section, Table 1 defines the game in precise mathematical detail, while the text discusses, interprets and justifies each phase of the game laid out in Table 1. Readers particularly interested in game theory may begin with Table 1 to see how my interpretation of the game, as detailed in the second and third subsections, results from its definition. The general reader can follow along in the text to see how the mathematical definition of the game follows from the interpretation.

3.1 The Agents

The model has three types of agents: 1) a single profit maximizing *firm* or online merchant, 2) a unit mass of utility-maximizing *consumers* with different reservation prices, indexed by i from the set I, and 3) a single online *advertisement platform* that maximizes clicking.[8]

The firm sells a single good over the internet for a single price p of its choosing and faces a constant marginal cost c of production. The firm buys potential customers from the advertisement platform through pay-per-click pricing. This means that the firm pays the online advertisement platform a constant fixed pay-per-click advertising price a for each consumer that clicks on its ad.

Every consumer has a reservation price r for the product. Before clicking on the ad, the consumer does not know its reservation price because the consumer does not know anything about the product. Yet, the advertisement platform knows r. Although this might seem strange, consider the fact that the advertising platform is the only agent that has information about both the consumer and the product. For example, I do not know how much I am going to like a random Gershwin album about which I have no information; I do not know what songs are performed and by whom. I have an idea about the distribution of my reservation prices for Gershwin albums based on my observation of random albums in the past but no information about this particular album. The studio that created the album has invested a great deal to figure out the distribution of music-lover tastes but has no information about me personally. Yet, Google knows what albums I have searched for in the past and whether other people with similar tastes have bought this album.

If the consumer clicks on the ad, he pays a constant fixed search cost b and has the opportunity to buy the product. When deciding whether or not to click, the consumer uses two sources of information: 1) the fact that he has been shown the ad and 2) the rationally expected price of the firm.

The advertising platform is maximizing clicks on the ad. If there is not enough expected benefit from clicking on the ad, the advertising platform can target advertise to encourage consumers to click on the ad. The platform target advertises by choosing the proportion of consumers with each reservation price that is shown the ad. It does this to change the distribution of reservation prices of those consumers shown the ad. This changes the consumers expected benefit from clicking on an ad and the firms profit from each price.

[8] I am exploring the simple example where the pay-per-click pricing is fixed. The advertising platform maximizes profit by maximizing clicking.

3.2 Summary of the Game

First, each consumer draws his reservation price r, which is only observed by the advertising platform. Second, the advertising platform target advertises by choosing the proportion of consumers with each reservation price that is shown the ad; this proportion is observed by all agents. Third, simultaneously the firm chooses its single price p, and each consumer who is shown the ad decides whether or not to click on it. Fourth, all the consumers that clicked on the ad observe their reservation price r and the product price p. Fifth, each consumer who clicked on the ad decides whether or not to purchase the product. Sixth, all agents receive their payoffs.

The consumers who did not click on the ad get a payoff of zero, whether or not they were shown the ad. The consumers who clicked on the ad pay search cost b. I interpret the search cost as the forgone opportunities and energy that the consumer gives up to review the product details.[9] If a consumer buys the product, then he gets payoff $r-p$ in addition to paying b. The firm gets a payoff equal to its price markup, $p-c$, for every consumer who buys the product while paying a fixed pay-per-click price a for every consumer who clicks on his ad. The payoff to the advertising platform is the advertising revenue from the firm.

3.3 The Game

Table 1 defines the game. In this section, I will discuss and interpret the mathematical detail involve in each step of the game.

Phase 1, the *reservation price allocation phase*. A unit mass of consumers decided exogenously to visit the advertising platform's webpage.[10] Consumers who visit each draw a reservation price, their private value of the product; each consumer $i \in I$ draws but does not observe his reservation price r^i from an identical probability distribution with probability density function f, cumulative probability density function F and hazard function $h \equiv f/(1-F)$.

To provide a set of regular distributions of reservation prices, I assume that f is differentiable, nonzero and finite-valued over a closed and contiguous support along the real number line.[11] I assume that the hazard function h is strictly increasing or equivalently $1-F$ is log-concave.[12] Many commonly used probability distributions satisfy these conditions, including the normal, uniform, Pareto and logistic distributions.[13]

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[9]The consumer was not already looking for a product when the ad popped up. I am not modeling the decision to surf the net. I am modeling the decision to click on an ad. The decision to surf is exogenous.

[10]This is not a model of the decision to visit a webpage. I am assuming there is no additional search cost in surfing the web. The model could later be easily extended to incorporate this choice and cost.

[11]That is f has a support of the form $[l, u]$, $[l, \infty)$, $(-\infty, u]$ or $(-\infty, \infty)$ where l is a minimum and u is a maximum.

[12]This assumption will be sufficient to assure that the firm has a unique profit maximizing price p, if all consumers were shown the ad.

[13]See Bagnoli and Bergstrom (2005) for further examples.

Table 1: The Game

	Phase	**Actions**	**Who Observes What**
1.	reservation price allocation	Each consumer i draws r^i *i.i.d.* with pdf f, cdf F and hazard function $h \equiv f/(1-F)$	·The only agent to observe r^i is the ad platform. ·The distribution of f is common knowledge.
2.	targeting decision	·The ad platform chooses a function $m : \Re \Rightarrow \Re$ such that $m(p) \in [0, f(p)]$ for all p. ·Each consumer i is shown the ad with probability $m(r^i)/f(r^i)$.	Everyone observes m and who is shown the ad.
3.	clicking and pricing decision	Each consumers who is shown the ad decides whether or not to click on the ad. The firm chooses the price p.	·Everyone observes who clicks on the ad. ·Only the firm observes its price p.
4.	price reassurance and reservation price revelation	–	Each consumer who clicked on the ad observes the price p and his reservation price r^i.
5.	sales	Each consumer who clicked on the ad decides whether or not to buy the product.	Everyone observes who buys the product.
6.	payoffs	·Each consumer i suffers the cost b if he clicked on the ad and gains the utility $(r-p)$ if he bought the product. ·The firm suffers a for each consumer who clicked on the ad and gains $p-c$ from each consumer that bough its product. ·The ad platform gains a for each consumer who clicked on the ad.	–

While the distribution of reservation prices is common knowledge, only the advertising platform knows the reservation price of each consumer. The advertising platform is the only agent with information about both the consumer and the product. At this point, the consumer knows nothing about the product, and the firm knows nothing about the consumer, so neither the firm nor the consumer knows the reservation price. The advertisement platform has gathered enough information about the product and the consumer to know each consumer's reservation price.

Phase 2, the *targeting decision phase*. The advertising platform chooses the proportion of consumers with each reservation price to whom the ad is shown to maximize clicking; the advertising platform chooses the density of consumers, $m(p) \in [0, f(p)]$, shown the ad out of the density of consumers $f(p)$ looking at the website for every possible reservation price $p \in \Re$ to maximize the quantity $\overline{M}_c$ of consumers clicking on the ad. The advertising platform chooses a *targeting strategy*, which I define as a function $m : \Re \Rightarrow \Re$ satisfying conditions CM1-4 (conditions CM1, CM2, CM3 and CM4).

CM1 $m(p) \in [0, f(p)]$ for all $p \in \Re$.

CM2 m does not have essential discontinuities, removable discontinuities and infinite number of jump discontinuities.

CM3 m is rightwise continuous at the infimum of the set inputs that give nonzero values.

CM4 m is leftwise continuous at all other values.

Because the advertising platform should not be able to show the ad to more consumers than exist, I restrict the platform's choice of m by $m(p) \in [0, f(p)]$ for all p [CM1]. For example, if $m(\$2) = .12$ and $f(\$2) = .24$, then half of the consumers who would be willing to pay up to and no more than \$2 for the product would be shown the ad. Yet if somehow $m(\$2) = .46$, then the advertising platform would be showing the ad to twice as many consumers with a reservation price of \$2 than exists.

Because I want the one-sided limits of m from both directions to exist, I do not allow the advertisement platform to choose targeting strategies m with essential discontinuities,[14] removable discontinuities,[15] or infinite an infinite number of "jump discontinuities" [CM2]. I restrict m to have a finite number of only one type of possible discontinuity, *jump discontinuities*;[16] I also restrict m to have different one-sided limits at a finite number of prices. If m has a jump discontinuity at price p, then it does not matter what value of $m(p)$ I choose.

[14]These are discontinuities that make it impossible to integrate. For example, there is an essential discontinuity at $x = 0$ when $f(x) = sin(1/x)$.

[15]These are discontinuities were the right-hand and left-hand limits exist, are finite and are equal. For example, there is a removable discontinuity at $x = 0$ when $f(x) = x/x$.

[16]These are discontinuities were the right-hand and left-hand limits exist and are finite. For example, there is a jump discontinuity at $x = 0$ when $f(x) = |x|/x$.

Changing the value of m at one price does not change the value of the cumulative function $M(p') \equiv \int_{-\infty}^{p'} m(p)dp$ for all p', nor of any other function with an output of mass of consumers as a function of price. I restrict the values of m at jump discontinuities to be the most convenient for my mathematical proofs: I restrict the advertisement platform's choice of m to be *rightwise continuous*[17] at the infimum of the set inputs that give nonzero values of m [CM3] and *leftwise continuous*[18] at all other values [CM4].

To aide my examination of the function m and simplify my notation, I define a few new variables and functions. The mass of all consumers shown the ad, the area under the curve m, is defined as $\overline{M} \equiv \int_{-\infty}^{\infty} m(p)dp$. The mass of all consumers with reservation prices less than or equal to p', m's cumulative function, is defined as $M(p') \equiv \int_{-\infty}^{p'} m(p)dp$ for all p'. m's hazard function is defined as $h_m \equiv m/(\overline{M} - M)$.

The way in which an advertisement platform's targeting strategy m determines which consumers get shown an ad is explained through the function $\omega : I \Rightarrow \{0, 1\}$. If $\omega^i = 1$, consumer i is shown the ad, and if $\omega^i = 0$, consumer i is not shown the ad. If $m(p) = f(p)$, then all consumers with a reservation price $r = p$ are shown the ad. If $m(p) = 0$, then no consumers with a reservation price $r = p$ are shown the ad. Otherwise, if $0 < m(p) < f(p)$, then each consumer i with reservation price $r = p$ is shown then ad with a probability $m(p)/f(p)$.

In order for targeted advertising exists, I assume that the advertising platforms choice of targeting strategy m is common knowledge. This is to disallow a credibility issue, where the advertising platform tells the firm and consumers it is using one targeting strategy while it is really using another. Because the advertisement platform is maximizing clicks, the advertisement platform would want to lie enough to get consumers to click on its ad and then show the ad to everyone. I will discuss the credibility of this assumption and how it could be relaxed in Section 9.

To keep my model simple, I assume that the ad reveals no information about the product to the consumer. This is similar to most of the advertising literature. I showed at the outset of Section 3 that targeted advertising shares a similar purpose as limited advertising content. Because the consumers know the targeting strategy, the fact that the advertising platform shows a consumer the ad gives that consumer some information about how much he will like that product.

Phase 3, the *clicking and pricing decision phase*. Here the firm chooses its single price p, and each consumer who is shown the ad chooses whether or not to click, simultaneously and independently. To simplify notation, I define consumer i's clicking strategy as his choice of $\omega_c^i \in \{0, 1\}$. If consumer i chooses to click on the ad, then he chooses $\omega_c^i = 1$. If i chooses not to click, then he chooses $\omega_c^i = 0$.

When a consumer clicks on the ad, he commits to spend all the time and effort required to look at the product details, and he commits the firm to pay

[17] continuous from the positive direction

[18] continuous from the negative direction

the pay-per-click advertising price to the advertising platform. I define the consumer's forgone opportunities and energy spent reviewing the product as the *search cost*, with a fixed value b,[19] and the pay-per-click price as the fixed value a.

If a consumer is not shown the ad, then he can not click on the ad, so his value ω_c is automatically zero. To obtain a well-defined equilibrium, I assume that a consumer who is indifferent between clicking and not clicking will click, and a firm that is indifferent between setting more than one price will set the lowest price.

To aide the examination of clicking behavior and simplify my notation, I define the following. The density of consumers with reservation price p clicking on the ad is defined as the function $m_c(p) : \Re \Rightarrow \Re$, such that $m_c(p)/f(p)$ is the probability that an arbitrary consumer with a reservation price of p has a value $\omega_c = 1$. The quantity of consumers clicking, or in other words the area under the curve m_c, is $\overline{M}_c \equiv \int_{-\infty}^{\infty} m_c(p)dp \leq \overline{M}$. The mass of all consumers with reservation prices less than or equal to p clicking on the ad is the cumulative function $M_c(p') \equiv \int_{-\infty}^{p'} m_c(p)dp$ for all p. The hazard function is $h_{m_c} \equiv m_c/(\overline{M}_c - M_c)$.

Phase 4, the *price reassurance and reservation price revelation phase.* Each consumer who clicked on the ad now sees the product price p and his reservation price r. In equilibrium, the product price that the consumers rationally expect will be the actual price. Hence, the revelation of the price can be seen as a reassurance of the price.

Phase 5, the *sales phase.* Here the consumers who clicked on the ad choose whether or not to buy the product. To simplify notation, I define consumer i's buying strategy as his choice of $\omega_b \in \{0, 1\}$, with $\omega_b = 1$ indicating that consumer i chooses to buy the product and $\omega_b = 0$ indicating that consumer i chooses not to buy the product. A consumer will buy the product when his reservation price r exceeds product price p. Arbitrarily, if a consumer is indifferent between buying and not buying so $r = p$, then the consumer will buy the product. If a consumer does not click on the ad, then he cannot buy the product, so his value ω_b is automatically zero.

I define the *mass of consumers buying* or equivalently the *quantity of the product* sold as $Q \equiv \int_{i \in I} \omega_B^i di$. If $Q = 1$, then all consumers buy the product, and, if $Q = 0$, then no consumers buy the product.

Phase 6, the *payoff phase.* Here all the agents get their payoffs. Consumers who clicked on the ad pay the search cost b. Consumers who buy the product get surplus from their difference $r - p$, so each consumer i gets a payoff of $u^i \equiv (r^i - p)\omega_B^i - b\omega_c^i$. The firm gets its price markup $p - c$ for the quantity Q of the product it sells and pays pay-per-click price a for the quantity $\overline{M}_c$ of consumer clicks. So, the firm gets a payoff of $\Pi \equiv (p - c)Q - a\overline{M}_c$. The advertising platform gets a payoff of the advertising revenue $A \equiv a\overline{M}_c$ paid by the firm.

[19] The consumer was not already looking for a product when the ad popped up. I am not modeling the decision to surf the net. I am modeling the decision to click on an ad. The decision to surf is exogenous.

4 Each Consumer's Clicking Decision

Now that I have defined each phase of the game, I will recursively solve for the equilibrium through backwards induction. In the last section, I found that a consumer that has clicked on the ad will buy the product if his reservation price r is greater than or equal to the price of the product p. In this section, I solve for how a consumer that is shown the ad would decide to click on the ad. In the next section, I will analyze the firm's pricing decision. From these two sections I will be able to aggregate all the decisions made in Phase 3, the clicking and pricing decision phase. Then in Section 6, I will solve the equilibrium by solving for which consumers the advertising platform would show the ad to.

A consumer that is shown the ad would choose to click on the ad if it increases his expected payoff. To calculate his expected payoff, a consumer has three pieces of information: 1) the rationally expected price, 2) the fact that he is shown the ad and 3) the advertising platforms targeting strategy m. From this information he is able to infer the equilibrium price p^* through rational expectations and a distribution of his possible reservation prices $m/\overline{M}$.[20]

As I mentioned earlier, a consumer will buy the product if his reservation price r exceeds the product price p^*. Therefore if a consumer is shown the ad, then his expected benefit b_c from clicking on the ad is given by equation (1).

$$b_c \equiv E[max\{r - p^*, 0\}|\omega = 1] = \int_{p^*}^{\infty} (p - p^*)m(p)dp/\overline{M} \tag{1}$$

A consumer's expected payoff from clicking is his expected benefit b_c minus his search cost b. Thus consumers will click following the *consumers' clicking condition* given below.

Consumers' Clicking Condition. *A consumer who is shown the ad will click on the ad if and only if $b_c \geq b$, or equivalently $\omega_c = \omega * 1\{b_c \geq b\}$.*

This is an identical decision for all consumers that are shown the ad. Either all consumers that are shown the ad click or none click. This is given by equation (2).

$$m_c = m * 1\{b_c \geq b\} \tag{2}$$

Here, m_c is the density function of consumers clicking, and m is the density function of consumers shown the ad. They are equal when the consumers' clicking condition is met.

5 The Firm's Pricing Decision

In this section, I solve for the firm's pricing decision. This occurs simultaneously with the consumer's clicking decisions. In the following section, I will use

[20] Recall that the area under the function m is $\overline{M} \leq 1$. Therefore weighting m by $1/\overline{M}$ makes a probability density function–the probability density function of a random reservation price from those consumers shown the ad.

the conditions I found in both sections to solve for the advertising platform's targeting decision and solve the equilibrium.

The firm chooses its price p^* to maximize its profit $(p-c)Q - a\overline{M}_c$. Because the firm takes the advertising cost $a\overline{M}_c$ as a constant, the firm chooses its price p^* to maximize its profit $(p-c)Q$ from sales.

The firm takes the density function m of consumers shown the ad and infers its demand function Q. It does this through connecting m to the density function m_c of consumers clicking and then connecting m_c to the demand function, Q. If only a fraction θ of consumers were to click on the ad, then $m_c = \theta m$, because the consumers would not know their reservation price when clicking on the ad. Because all the consumers who click with a reservation price r weakly exceeding the product price p^* will buy the product, then $Q(p^*) = \int_{p^*}^{\infty} m_c(p)dp = \overline{M}_c - M_c(p^*) = \theta[\overline{M} - M(p^*)]$ for all p^*. Therefore, the firm sets prices according to the *firm's profit maximizing condition* given below.[21]

Firm's Profit Maximizing Condition. *The firm will choose its price p^* such that $p^* = \arg\max_p (p-c)(\overline{M} - M(p))$.*

If m is continuous at p^*, then the firm's profit maximizing condition is equivalent to the first-order-condition given in equation (3).

$$(p^* - c)m(p^*) = \overline{M} - M(p^*) \tag{3}$$

This equation is the standard Bertrand profit maximization first-order conditions for the demand curve $\overline{M} - M$.

6 The Ad Platform's Targeting Decision

In this section, I analyze how the advertising platform chooses the proportion $m(p)$ of consumers with each reservation price $r = p$ to whom the ad is shown, given the consumer and firm strategies discussed above.

Because the pay-per-click price a is a fixed constant, the advertising platform chooses its targeting strategy to maximize the mass $\overline{M}_c$ of consumers clicking. The advertising platform knows the consumers' clicking condition and the firm's profit maximizing condition, so the advertising platform is able to rationally expect who will click and the price p^* set by the firm under any choice of targeting strategy m.

The expected value b_c of clicking on an ad can be written as a function of the targeting strategy m. And, by equation (2), the mass $\overline{M}_c$ of consumers clicking is a function of the targeting strategy m which I define by equation (4).

$$M_c = \overline{M} * 1\{b_c(m, p^*(m)) \geq b\} \tag{4}$$

[21] I realize that if $\theta = 0$, any price would maximize profit. I assume that the firm arbitrarily chooses the price that maximizes the profit maximizing condition. This rules out the uninteresting degenerate equilibrium where the firm sets its price really high and none of the consumers click on the ad.

Here $\overline{M}$ is the mass of consumers shown the ad and p^* is the price that would be set under the firm's profit maximizing condition.

Equation (4) is the maximization problem that the advertising platform solves. The advertising platform will choose a targeting strategy m that maximizes the mass $\overline{M}_c(m)$ of consumers clicking.

6.1 A Benchmark Result: Advertise to All Consumers

In this first subsection, I analyze when the advertising platform would show the ad to all consumers. Here, the function m would equal the probability density function f, so by the profit maximizing condition, the firm would set the standard monopoly price $p^m \equiv \arg\max_p (p - c)(1 - F(p))$. Because the hazard function h is continuous and increasing, p^m exists and is unique.

Anticipating this price outcome, consumers would click on the ad if the expected benefit b_c from clicking on the ad were greater than the search cost b. This happens when the consumers' clicking condition is met and simplifies to the condition given in equation (5).

$$\int_{p^m}^{\infty} (p - p^m) f(p) dp \equiv b_1 \geq b \tag{5}$$

If this condition is met, then the advertising platform would want to advertise to everyone because doing so gets all consumers to click on the ad, yielding the maximum possible mass of clicking consumers, $\overline{M}_c = 1$. Because of the limits I set on the possible discontinuities in the function m, this solution is unique.[22]

The advertising platform could also optimally show the ad to all consumers if no targeting strategy exists that could ever induce any consumers to click on the ad. In this case any targeting strategy is optimal.

6.2 Preliminary Results: Advertise to Some Consumers

Next, I develop some preliminary results to examine the case when the advertising platform would show the ad to some but not all consumers. I will use my results from this subsection and the Subsection 6.1 to solve for the equilibrium in Subsection 6.3. When showing the ad to all consumers, $m = f$, would not induce any consumers to click, while some other targeting strategy $\widetilde{m}$ would induce some consumers to click, then the advertising platform would optimally show the ad to some but not all consumers. Formally this is shown by Lemma 2.[23]

Lemma 2. *If $\int_{p^m}^{\infty} (p - p^m) f(p) dp \equiv b_1 < b$ and there exists a targeting strategy $\widetilde{m}$ satisfying CM1-4 that would induce consumers to click, then the advertising platform will not advertise to every consumer.*

[22] This is formalized in Lemma 1. See Appendix A for a formal proof.
[23] See Appendix A for a formal proof.

Now I examine whom the advertising platform would show the ad to when it shows the ad selectively. To do this, I examine, in parts, the function m that the advertising platform would choose. I start by examining the function m at values below the rationally expected price p^*, when the platform shows the ad to consumers with reservation prices $r < p^*$. These are the consumers who the advertising platform shows the ad but are rationally expected not to buy the product.

To do so, I consider two targeting strategies, $\widetilde{m}$ and m, that are the same function above the rationally expected price $\widetilde{p}$ of the targeting strategy $\widetilde{m}$. Given the firm's profit maximizing condition, changing the targeting strategy from $\widetilde{m}$ to m can only induce the firm to lower its price and never induce it to raise its price. Therefore changing the targeting strategy from $\widetilde{m}$ to m would only change the expected benefit b_c from clicking by: 1) changing the mass of consumers shown the ad, and 2) decreasing the rationally expected price. If the targeting strategies $\widetilde{m}$ and m were to have the same mass of consumers clicking (i.e. $\overline{M} = \overline{\widetilde{M}} \equiv \int_{-\infty}^{\infty} \widetilde{m}(p)dp$) then changing targeting strategy from $\widetilde{m}$ to m could only increase b_c through decreasing rationally expected price. This is the general idea behind the proof of Lemma 3.[24]

Lemma 3. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and either (a), (b) or (c) that produces weakly more clicking.*

a) $m(p) = f(p)\ \forall p \leq p^*$

b) $m(p) = f(p)\ \forall p \in [\underline{p}, p^*]$ *and* $m(p) = 0 \forall p < \underline{p}$

c) $m(p) = 0\ \forall p \leq p^*$

where p^* is the price set by the firm under m and $\underline{p} \in (-\infty, p^*)$.

Because targeting strategies like those in Lemma 3 always produce weakly more clicking than any other targeting strategy, a click-maximizing targeting strategy exists of this form. Lemma 3 reduces the advertising platform's choice of targeting strategy below the rationally expected price p^* to a one-dimensional cutoff price $\underline{p}$, where $\underline{p} \to -\infty$ under (a), $\underline{p} \in (-\infty, p^*)$ under (b) and $\underline{p} = p^*$ under (c).

For example, if $f(p) = 1\{0 \leq p \leq 1\}$ and $c = 0$, then the ad platform would prefer the targeting strategy $m(p) = 1\{1/2 \leq p \leq 1\}$ to the targeting strategy $\widetilde{m}(p) = 1\{2/3 < p \leq 1\} + 1/4 * 1\{0 \leq p \leq 2/3\}$. Both targeting strategies m and $\widetilde{m}$ show the ad to $\overline{M} = 1/2$ consumers. Both targeting strategies m and $\widetilde{m}$ produce the same demand curve $\overline{M} - M(p) = 1 - p$ above the rationally expected price $\widetilde{p} = 2/3$ of targeting strategy $\widetilde{m}$. Yet choosing strategy m would induce the firm to lower its price to $p^* = 1/2$ and thus increase the expected benefit b_c from clicking on the ad from 5/36 to 1/4. If consumers were willing to click under targeting strategy $\widetilde{m}$, then the search cost $b \leq 5/36$, so consumers would

[24] See Appendix A for a formal proof.

click under m. Yet if the search cost $5/36 < b \leq 1/4$, then consumers would be willing to click under targeting strategy m, but not under targeting strategy $\widetilde{m}$. Lemma 3 claims that the ad platform would prefer targeting strategies like m to targeting strategies like $\widetilde{m}$, because they weakly induces more consumers to click.

Now that I have examined how an ad platform would advertise to consumers with reservation prices r less than or equal to the rationally expected price p^*, I consider how an ad platform would advertise to consumers with reservation prices $r = p^*$. Later, I will combine these to figure out how an advertising platform will target advertisements.

I find that there is always a weakly preferable targeting strategy for the advertising platform such that $m(p^*) = f(p^*)$. If a targeting strategy $\widetilde{m}$ with a rationally expected price $\widetilde{p}$ satisfies $\widetilde{m}(\widetilde{p}) < f(\widetilde{p})$, then the advertising platform could show the ad to more consumers in a way that encourages the firm to lowers its price, increasing the expected benefit b_c from clicking. This would increase clicking, as is shown formally by Lemma 4.[25]

Lemma 4. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and $m(p^*) = f(p^*)$ that produces weakly more clicking, where p^* is the price set by the firm under m.*

Lemma 4 relates to the firm's profit maximizing first-order condition from equation (3). The advertising platform would want to make $m(p^*)$ as big as possible so that the firm would sell to as many consumers as possible. Increasing the number of consumers who buy the product in this way increases each consumer's expected benefit from clicking.

For example, suppose $c = \$10$, and there was a unit mass of consumers with reservation prices distributed uniformly between \$0 and \$50. If the ad platform were to choose the targeting strategy $\widetilde{m}(p) = .1 * 1\{p \geq 30\}$, then the firm would sell its product for $\$30/unit$. If instead the ad platform were to choose the targeting strategy $m(p) = .1 * 1\{p < 30\} + .2 * 1\{29.5 \leq p \leq 30\}$, then the firm would sell its product for $\$29.5/unit$ to increase its profit by \$.07. More consumers would be shown the ad at a higher expected benefit b_c from clicking. Thus, if the consumers would click under $\widetilde{m}$, then they would click under m. Lemma 4 claims that the ad platform would always advertise to all the consumers with reservation prices equal to the rationally expected price p^*.

Lastly, I examine how an ad platform would advertise to consumers with reservation prices r greater than the rationally expected price p^*. I consider two targeting strategies, $\widetilde{m}$ and m, satisfying: 1) they are both the same function at and below the rationally expected price $\widetilde{p}$ of $\widetilde{m}$, 2) the firm weakly prefers price $\widetilde{p}$ to all other prices $p > \widetilde{p}$ under m and 3) they both have the same mass of consumers clicking (i.e. $\overline{M} = \overline{\widetilde{M}} \equiv \int_{-\infty}^{\infty} \widetilde{m}(p)dp$). The first and second conditions induce the firm to set price $p^* = \widetilde{p}$ under the targeting strategy m. The third condition simplifies my analysis of the effect on the expected benefit b_c of clicking by holding constant the mass of consumers who click.

[25]See Appendix A for a formal proof.

For any targeting strategy $\widetilde{m}$, there are potentially many other targeting strategies, m_1, m_2, m_3, ..., that meet these three conditions. So, the advertising platform might as well choose the targeting strategy m with the highest expected benefit b_c from clicking. If the consumers would click under any other targeting strategy $\widetilde{m}$ that has the rationally expected price $\widetilde{p}$ then the consumer would click under m. This is the idea behind the proof of Lemma 7.[26]

Lemma 7. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and either (a), (b) or (c) that produces weakly more clicking.*

a) $m(p) = f(p)\ \forall p > p^*$

b) $m(p) = K/(p-c)^2\ \forall p \in (p^*, \widehat{p}]$ *and* $m(p) = f(p)\ \forall p > \widehat{p}$

c) $m(p) = K/(p-c)^2\ \forall p > p^*$

where p^* is the price set by the firm under m, $K \equiv f(p^*)(p^*-c)^2$ and $\widehat{p} \in (p^*, \infty)$.

The targeting strategies in Lemma 7 show the ads to maximize the expected benefit b_c from clicking for consumers without changing the mass of consumers or inducing the firm to increase its price. It does this by showing the ad to as few consumers as possible below some cutoff price $\widehat{p}$ and too as many consumers as possible above $\widehat{p}$. Showing the ad to fewer consumers below $\widehat{p}$ would induce the firm to increase its price.

For example, suppose $c = \$3$ and there was a unit mass of consumers with reservation prices distributed uniformly between \$3 and \$15. If the ad platform were to show the ad to all consumers with reservation prices between \$4 and \$14, $\widetilde{m}(p) = \frac{1}{12} * 1\{4 \leq p \leq 14\}$, then the firm would sell its product for $\$9/unit$. If the ad platform were to show the ad using targeting strategy $m(p) = \frac{1}{12} * 1\{4 \leq p \leq 9\} + 3/(p-3)^2 * 1\{9 < p \leq 12\} + \frac{1}{12} * 1\{12 < p \leq 15\}$, then the firm would still sell its product for $\$9/unit$, and the ad platform would still show the ad to the same mass of consumers, $\overline{M} = 5/6$. But, m has a higher expected benefit b_c from clicking than $\widetilde{m}$. Therefore, if consumers would click under $\widetilde{m}$, they would click under m. Thus, Google weakly prefers m to $\widetilde{m}$.

Now that I have examined the function m piecemeal, I bring together my conclusions from Lemmas 3, 4 and 7 to get Lemma 8, showing how an advertisement platform would target-advertise to all consumers.

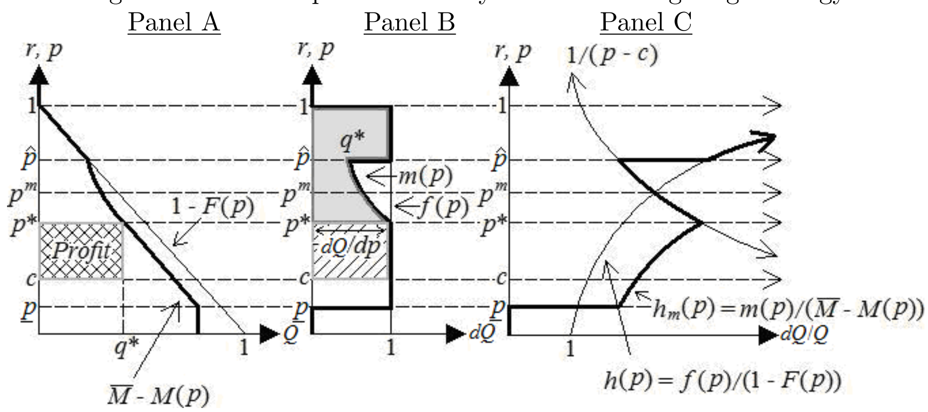
Lemma 8. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and either (a), (b), (c) or (d) that produces weakly more clicking.*

a) $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$

b) $m(p) = f(p) * 1\{p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$

[26] See Appendix A for a formal proof. Lemmas 5 and 6 in Appendix A are used to prove Lemma 7.

Figure 1: An Example of a Weakly Preferable Targeting Strategy



c) $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p-c)^2 * 1\{p > p^*\}$

d) $m(p) = f(p) * 1\{p \leq p^*\} + K/(p-c)^2 * 1\{p > p^*\}$

where p^* *is the rationally expefect prices under* m*,* $K \equiv f(p^*)(p^* - c)^2$*,* $\widehat{p} \in [p^*, \infty)$ *and* $\underline{p} \in (-\infty, p^*]$*.*

The advertising platform would always weakly prefer one of the targeting strategies in Lemma 8 to any other targeting strategy. Note that targeting strategies of the forms (b), (c) and (d) are the limits of serieses of targeting strategies of the form (a). Lemma 8 reduces the advertising platforms choice of targeting strategy to two choice variables: $\underline{p}$ and $\widehat{p}$. Choosing $\underline{p} \to -\infty$ is the same as choosing a targeting strategy of the form (b) or the form (d). Choosing $\widehat{p} \to \infty$ is the same as choosing a targeting strategy of the form (c) or the form (d).

Figure 1 illustrates the targeting strategies shown in Lemma 8 for a uniform distribution. The dark lines are the various forms of the targeting strategy m in different spaces: 1) the demand curve $\overline{M} - M$, 2) the density function m and 3) the hazard function h_m. In panel A, I compare the Demand Curve $\overline{M} - M$ to the function $1 - F$, the demand curve that the firm would face if all consumers were shown and subsequently clicked on the ad. In panel B, I compare the density functions m and f in order to see the density of consumers shown the ad at every reservation price. In panel C, I compare hazard functions h_m and h in order to see the nice mathematical properties of the hazard function.

To rationally expect the price that the firm would set, the advertising platform uses the firm's profit-maximizing condition. Panel A illustrates this by having the firm pick its price p^* to maximize $(p - c)Q$, given by the cross-hatched region. The firm's first-order condition from equation (3) indicates how it would maximize its profit in Panel B. The firm would equate the marginal loss $(p - c)m(p)$ from raising its price (i.e. the diagonal-striped box) with the marginal gain $\overline{M} - M(p)$ from raising its price (i.e. the grey region). Panel C illustrates how to find the firm's price through the hazard function condition $h_m(p^*) = 1/(p^* - c)$, which is equivalent to the firm's first-order condition given by equation (3). The firm will set its price where the hazard function h_m crosses the curve $1/(p - c)$.

The targeting strategy given in Figure 1 gives the firm equal profit for setting any price between p^* and $\widehat{p}$. The cross-hatched box of the firm's before-advertising expense profit $(p - c)Q$ in Panel A is equal and maximized for any price $p \in [p^*, \widehat{p}]$. Thus the demand curve between prices p^* and $\widehat{p}$ is the familiar demand curve of constant profit $Q(p) = \overline{M} - K/(p - c)$. In Panel B, the box $(p - c)m(p)$ of marginal profit lost from raising the price is equal to the grey region $\overline{M} - M(p)$ of marginal profit gained for any price $p \in [p^*, \widehat{p}]$. This produces the curve between p^* and $\widehat{p}$. In Panel C, the hazard function follows along the curve $1/(p - c)$ when $p \in [p^*, \widehat{p}]$ because all of these prices maximize profit.[27]

For reservation prices above $\widehat{p}$, the advertising platform shows the ad to all consumers. Therefore in Figure 1, the demand curve, density function and hazard function above $\widehat{p}$ is no different than they are when the advertising shows the ad to all consumers.

For reservation prices below the rationally expected price p^* and above the price cutoff $\underline{p}$, the advertising platform shows the ad to all consumers. This is shown in panel A where the demand function $\overline{M} - M$ follows parallel to $1 - F$, because the quantity demanded at every price between $\underline{p}$ and p^* is lower by a constant amount: the mass of consumers not shown the ad with reservation prices above the price p^*. This is shown in Panel B by having $m = f$. These combine to shift out the hazard function h_m from the hazard function h by as shown in Panel C.

For reservation prices below the price cutoff $\underline{p}$, the advertising platform does not show the ad to any consumers. Thus in Panel A, the quantity demanded $\overline{M} - M$ would remain constant and in Panels B and C the density and hazard functions would be zero.

6.3 Optimal Advertising

I now build upon the previous two subsections to find an optimal targeting strategy for any search cost b and any marginal cost of production c. I start by solving for an optimal targeting strategy when showing the ad to all consumers does not induce consumers to click using subsection 6.2. Then, I combine it with my conclusions from my benchmark results from section 6.1.

Because the advertising platform internalizes the firm's pricing choice and the consumers' clicking choices when it chooses its targeting strategy m, I also

[27]Note that the condition that a firm that is indifference between a set of prices would always pick the lowest is playing a role here by making the firm choose the price p^*. Without this condition, the ad platform could choose a strategy close to this strategy to guarantee the firm choose the lowest price, by giving the firm slightly more profit from choosing p^*. The problem with this is that for any close strategy, the ad platform can choose a strategy slightly closer to gain slightly more clicks.

find an equilibrium of the game for any search cost b and any marginal cost of production c. In Section 7 I will examine how the payoffs depend on b and c.

First, I consider how the consumers' clicking condition interacts with the advertising platform's strategy. When advertising to some but not all consumers, the ad platform will choose a targeting strategy m such that the consumers will have an expected benefit b_c from clicking on the ad equal to their search cost b. If they expected any lower benefit, then it would not be worthwhile to click. If they expected any greater benefit, then the advertisement platform could advertise to slightly more consumers and still induce consumers to click on the ad. This is formally given by Lemma 9.[28]

Lemma 9. *If $\int_{p^m}^{\infty}(p-p^m)f(p)dp \equiv b_1 < b$ and there exists a targeting strategy $\widetilde{m}$ satisfying CM1-4 that would induce consumers to click, then the advertising platform would choose a targeting strategy such that $b_c = b$.*

For example, suppose $f(p) = \{0 \le p \le 1\}$, $c = 0$, and $b = 3/16$. If all consumers were shown the ad (i.e. the targeting strategy $m_1(p) = 1\{0 \le p \le 1\}$), then consumers would not click on the ad, because the expected benefit $b_c = 1/8$ from clicking was less than the search cost $3/16$. If instead the ad platform were to show the ad to all consumer with reservation prices $r \ge 1/2$ (i.e. the targeting strategy $m_2(p) = 1\{1/2 \le p \le 1\}$), then all consumers shown the ad would click, because $b_c = 1/4 > 3/16$. Lemma 9 claims that the ad platform would show the ad to even more consumers than those shown the ad in m_2. For example, the ad platform could show the ad to all consumer with reservation prices $r \ge 1/3$ (i.e. the targeting strategy $m_3(p) = 1\{1/3 \le p \le 1\}$), then all consumers shown the ad would click, because $b_c = 3/16$ which equals the search cost b.

Lemma 9 relates to those strategies given in Lemma 8 through the advertising platform's choice variables $\underline{p}$ and $\widehat{p}$. If consumers get an expected benefit b_c from clicking that is strictly greater than their search cost b, then the platform can lower $\underline{p}$ or $\widehat{p}$ or both to increase the mass of consumers $\overline{M}$ who see the ad while still inducing all the consumers who see the ad to click.

Now that I have found that the expected benefit b_c from clicking will equal the search cost b, given the strategy that the platform will choose, I will solve for the ad platform's optimal targeting strategy. Given a search cost b and a feasible choice of the price cutoff $\underline{p}$, there is only one optimal choice of the variable $\widehat{p}$: the lowest $\widehat{p}$ that would yield the expected benefit b_c equal to the search cost b. Yet given any search cost b, there are potentially infinite combinations of $(\underline{p}, \widehat{p})$ that make $b_c = b$. I will now find exactly how the advertising platform will set $\underline{p}$ and $\widehat{p}$ to maximize clicking.

For example, if $f(p) = \{0 \le p \le 1\}$ and $c = 0$, then the consumer surplus at the rationally expected price p^* would be $CS(p^*(\widehat{p})) \equiv \int_{p^*(\widehat{p})}^{\infty}(p - p^*(\widehat{p}))m(p)dp = .5*(1-\widehat{p}^2)+\widehat{p}(1-\widehat{p})(.5ln(\widehat{p}) - .5ln(1-\widehat{p})-1)$, and the mass of consumers shown the ad would be $\overline{M}(\underline{p}, \widehat{p}) = 2\widehat{p}(1-\widehat{p}) - \underline{p}$. Note that $b_c(\underline{p}, \widehat{p}) = CS(p^*(\widehat{p}))/\overline{M}(\underline{p}, \widehat{p})$ $\forall \underline{p}, \widehat{p}$. If $CS(p^*(\widehat{p}))/\overline{M}(\underline{p}, \widehat{p}) = CS(p^*(\widehat{p}'))/\overline{M}(\underline{p}', \widehat{p}')$, then the consumers would

[28]See Appendix A for a formal proof.

have the same expected benefit from clicking on the targeting strategy with $(\underline{p}, \widehat{p})$ as they would with the targeting strategy with $(\underline{p}', \widehat{p}')$.

I find that, as the search cost b increases, the advertising platform would first increase the benefit from clicking on the ad by increasing $\widehat{p}$ up to some maximum $p_1 > p^*$, then by increasing $\underline{p}$ up to the rationally expected price p^* and then by increasing $\widehat{p}$ up to some maximum $p_2 > p_1$. The advertising platform shows ads in this way because initially increasing $\widehat{p}$ adds more expected benefit to clicking, with the same mass of consumers clicking lost as with an increase in $\underline{p}$; initially increasing $\widehat{p}$ also increases consumer surplus. After a while, it becomes more costly (in terms of consumers lost) for the advertising platform to increase $\widehat{p}$ and it switches to increasing $\underline{p}$. Eventually $\underline{p}$ cannot be increased any further and the advertising platform can still increase the benefit to clicking on the ad by increasing $\widehat{p}$. Ultimately increasing $\widehat{p}$ beyond some p_2 does not increase the expected benefit from clicking on the ad, so when the search cost b gets too high, no market exists for the product. This is formally shown in Proposition 1.[29]

Proposition 1. *There exists a click-maximizing targeting strategy m satisfying CM1-4 and the following:*

a) when $b \leq b_1$: $m = f$;

b) when $b_1 < b \leq b_2$: $m(p) = f(p) * 1\{p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ where $\widehat{p} \in (p^m, p_1]$;

c) when $b_2 < b \leq b_3$: $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq p_1\} + f(p) * 1\{p > p_1\}$ where $\underline{p} \leq p^*$;

d) when $b_3 < b \leq b_4$: $m(p) = K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ where $\widehat{p} \in (p_1, p_2]$;

e) when $b > b_4$: no market exists so any m is optimal;

for some $0 < b_1 < b_2 < b_3 < b_4$ where b is the search cost, p^* is the rationally expected price, $p_2 > p_1 > p^*$ and $K \equiv f(p^*)(p^* - c)^2$.

For example, if $f(p) = \{0 \leq p \leq 1\}$ and $c = 0$, then $p^m = .5$, $p_1 = \frac{2+\sqrt{2}}{4} \approx .85$, $p_2 \approx .89$ (p_2 solves $\frac{1+p_2}{p_2} = ln(\frac{p_2}{1+p_2})$), $b_1 = \frac{1}{8}$, $b_2 = \frac{1}{8} * (1 - \frac{1}{2+\sqrt{2}})^{-1} \approx .18$, $b_3 = \frac{1}{8}(1 - \frac{1}{2+\sqrt{2}} - \frac{\sqrt{2}}{4})^{-1} \approx .35$ and $b_4 \approx .35$ ($b_4 = \frac{\sqrt{(p_2(1-p_2)}}{2}[ln(\frac{p_2}{1-p_2}) - 1 + \frac{1}{p_2}] > b_3$). The firm chooses its price $p^* = \sqrt{\widehat{p}(1-\widehat{p})}$ and all consumers shown the will click if $b \leq b_4$.

7 Examination of Equilibrium Payoffs

In this section I examine the payoffs of the equilibrium where: 1) the ad platform chooses the click-maximizing targeting strategy found in Proposition 1,

[29]See Appendix A for a formal proof.

Table 2: Comparative Static Results of the Equilibrium Payoffs

<table>
<tr><th>Search Cost</th><th>Consumers</th><th>Ad Platform</th><th>High PPC Firm</th><th colspan="2">Low PPC Firm</th></tr>
<tr><td>$b < b_1$</td><td>$\frac{\partial E(u^i)}{\partial b} = -1$, $\frac{\partial E(u^i)}{\partial c} < 0$</td><td>$\frac{\partial A}{\partial b} = \frac{\partial A}{\partial c} = 0$</td><td colspan="3">$\frac{\partial \Pi}{\partial b} = 0$, $\frac{\partial \Pi}{\partial c} < 0$</td></tr>
<tr><td>$b_1 < b < b_2$</td><td rowspan="4">$\frac{\partial E(u^i)}{\partial b} = 0$, $\frac{\partial E(u^i)}{\partial c} = 0$</td><td rowspan="3">$\frac{\partial A}{\partial b} < 0$, $\frac{\partial A}{\partial c} < 0$</td><td rowspan="3">$\frac{\partial \Pi}{\partial b} > 0$, $\frac{\partial \Pi}{\partial c} > 0$</td><td>$\frac{\partial \Pi}{\partial b} < 0$</td><td rowspan="3">$\frac{\partial \Pi}{\partial c} < 0$</td></tr>
<tr><td>$b_2 < b < b_3$</td><td>$\frac{\partial \Pi}{\partial b} > 0$</td></tr>
<tr><td>$b_3 < b < b_4$</td><td>$\frac{\partial \Pi}{\partial b} < 0$</td></tr>
<tr><td>$b > b_4$</td><td>$\frac{\partial A}{\partial b} = \frac{\partial A}{\partial c} = 0$</td><td colspan="3">$\frac{\partial \Pi}{\partial b} = \frac{\partial \Pi}{\partial c} = 0$</td></tr>
</table>

2) the firm chooses its profit-maximizing price through equation (3) and 3) the consumers click when $b \leq b_4$. Table 2 summarizes my findings. I begin by examining a representative consumer's expected utility. Then I examine the ad platform's and the firm's profits.

7.1 The Consumers' Payoffs

By Lemma 9, if the search cost $b > b_1$, then a consumer's expected benefit b_c from clicking on the ad would be equal to b. Therefore his expected payoffs $E[u^i] = b_c - b$ from clicking would be equal to zero. Therefore as long as $b > b_1$, $\partial E(u^i)/\partial b = \partial E(u^i)/\partial c = 0$. Yet if $b < b_1$, then all consumers would choose click on the ad for any search cost b or for any firm's marginal cost c of production. Therefore as long as $b < b_1$, $\partial E(u^i)/\partial c < 0$ through lower prices and $\partial E(u^i)/\partial b < 0$.

7.2 The Ad Platform's Profits

If the consumers' search cost $b < b_1$, then the ad platforms profit A (i.e. the total ad revenue) would be equal to the pay-per-click price a times the total mass of consumers, one. Because both are exogenous, increasing or decreasing the consumers' search cost b or the firm's marginal cost c of production would not change the ad platform's profit A. If $b_1 < b < b_4$, then the ad platform's profit A would be equal to the mass of consumers shown the ad $\overline{M}$ times the pay-per-click price a. Because $\overline{M}$ is decreasing in search cost b and marginal cost c of production, so is the ad platform's profit A. Yet if $b > b_4$, then the ad platform cannot induce any consumers to click on the ad, so changing the search cost b or the marginal cost c of production would not affect the ad platform's profit A.

7.3 The Firm's Profits

If the consumers' search cost $b < b_1$, then firm sets the monopolists price p^m and its ad is shown to all consumers. Small changes in the consumers' search cost do not discourage or encourage more consumers from clicking on the ad or buying the product, so $\partial\Pi/\partial b = 0$. If the firm faced a higher marginal cost of production, $c' > c$, then the firm's profit Π would decrease, because by the Weak Axiom of Profit Maximization I have $(p^m(c) - c)Q(p^m(c)) - a > (p^m(c') - c)Q(p^m(c')) - a > (p^m(c') - c')Q(p^m(c)) - a$.

If the firm faces a high enough pay-per-click price a and $b_1 < b < b_4$, then the effect of changing the search cost b or the marginal cost c of production on the firm's profit Π would be overwhelmed by the effect on the ad revenue A. Because $A = a\overline{M}$ is decreasing in the search cost b and the marginal cost c of production, I have that $\partial\Pi/\partial b$, $\partial\Pi/\partial c > 0$. Paradoxically the firm would prefer a higher cost of production because the ad platform would choose to show the ad to fewer consumers.

If the firm faces a low enough pay-per-click price a and $b_1 < b < b_4$, then the effect of changing the search cost b or the marginal cost c of production on the firm's profit Π would be overwhelmed by its profit from sales $(p - c)Q$. Therefore a higher marginal cost c of production would decrease its profit Π. If $b_1 < b < b_2$ or $b_3 < b < b_4$, then a higher search cost b leads to a higher $\widehat{p}$, which leads to a lower price p^* and less profit Π. If $b_2 < b < b_3$, then a higher search cost b leads to a higher $\underline{p}$. The firm's profit from sales $(p - c)Q$ would not be affected and the firm faces a lower advertisement cost A, so $\partial\Pi/\partial b > 0$.

If the consumers' search cost $b > b_4$, then no market exists, so $\partial\Pi/\partial b = \partial\Pi/\partial c = 0$.

8 Take-it-or-leave-it Offer Advertising

One criticism of my model is that I take the pay-per-click price of advertising as an exogenous constant a. In this section I explore when the effect of the targeting strategy influences the price of advertising. I explore a simple adaptation to my model with an endogenous price of advertising. This adapted game is shown in Table 3. In this game the ad platform makes a take-it-or-leave-it advertising offer ϕ with its targeting decision. If the firm accepts this offer, then each consumer with a reservation price of $r = p$ is shown the ad with probability $m(p)/f(p)$ and the firm pays the ad platform ϕ for advertising (instead of paying $A = a\overline{M}$). If the firm rejects the offer, then no consumers are shown the ad, and the ad platform and the firm get a payoff of 0.

The firm would accept any offer $\phi \leq (p^* - c)(\overline{M} - M(p^*))$. The ad platform will make the largest take-it-or-leave-it offer that the firm would be willing to accept, so the ad platform will set its take-it-or-leave-it offer ϕ to the rationally anticipated $(p^* - c)(\overline{M} - M(p^*))$ and thus extract all the surplus from the firm. The ad platform will choose its targeting strategy m to maximize the offer ϕ that the firm would accept. Therefore the ad platform will choose a targeting

Table 3: The Take-it-or-leave-it Offer Game

	Phase	Actions	Who Observes What
1.	reservation price allocation	Each consumer i draws r^i *i.i.d.* with pdf f, cdf F and hazard function $h \equiv f/(1-F)$	·The only agent to observe r^i is the ad platform. ·The distribution of f is common knowledge.
2.	targeting and offer decision	·The ad platform chooses a function $m : \Re \Rightarrow \Re$ such that $m(p) \in [0, f(p)]$ for all p, and the ad platform chooses a take-it-or-leave-it offer $\phi \in \Re$.	Everyone observes m and the firm observes ϕ.
3.	take-it-or-leave-it decision	·The firm either accepts the offer or rejects the offer. ·If the firm accepts the offer, each consumer i is shown the ad with probability $m(r^i)/f(r^i)$. ·If the firm rejects the offer, then no consumer is shown the ad.	Everyone observes m and who is shown the ad.
4.	clicking and pricing decision	Each consumers who is shown the ad decides whether or not to click on the ad. The firm chooses the price p.	·Everyone observes who clicks on the ad. ·Only the firm observes its price p.
5.	price reassurance and reservation price revelation	–	Each consumer who clicked on the ad observes the price p and his reservation price r^i.
6.	sales	Each consumer who clicked on the ad decides whether or not to buy the product.	Everyone observes who buys the product.
7.	payoffs	·Each consumer i suffers the cost b if he clicked on the ad and gains the utility $(r-p)$ if he bought the product. ·If the firm accepted the offer, then the firm pays the ad platform ϕ and gains $p-c$ from each consumer that bough its product. ·If the firm rejected the offer, then the firm and ad platform get 0.	–

strategy that maximizes the profit $(p^* - c)(\overline{M} - M(p^*))$ of the firm. One such targeting strategy is given in Proposition 2.[30]

Proposition 2. *There exists a firm-profit-maximizing targeting strategy m satisfying CM1-4 and the following:*

a) when $b \leq b_5$: $m(p) = f(p) * 1\{p \geq p^m\}$;

b) when $b_5 \leq b \leq b_4$: $m(p) = K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ where $\widehat{p} \in (p^m, p_2]$;

c) when $b > b_4$: no market exists so any m is optimal;

for some $0 < b_5 < b_4$ where b is the search cost, p^* is the rationally expected price, $p_2 > p^m$ and $K \equiv f(p^*)(p^* - c)^2$.

The firm gets no profit from advertising to consumers that it would never sell to. Therefore the ad platform has no reason to advertise to these consumers when trying to maximize the value of a take-it-or-leave-it offer ϕ, as seen in Proposition 2. Note that for low enough search costs $b \leq b_3$, the advertising platform would advertise to some consumers that would not buy the product to increase the number of clicks on the ad, as seen in Proposition 1. Therefore when $b \leq b_3$, take-it-or-leave-it offer advertising is more efficient than pay-per-click advertising with fixed pay-per-click prices. Which pricing system best reflects reality? It would depend on the industry. In industries with a lot of competition for advertising, it is reasonable to expect that firms take the price of advertising as exogenous.[31]

Another interesting result of Proposition 2 is the ad platform's strategy when $b_5 \leq b \leq b_4$. Here the ad platform is not able to induce consumers to click on the ad with the targeting strategy $m(p) = f(p) * 1\{p \geq p^m\}$, so the ad platform chooses a targeting strategy that will induce the firm to charge a lower price. This is similar to the result in Anderson and Renault (2006). They explore endogenous advertising content in a costly search model. They find that when search cost are large enough, firms will commit to lower prices in their advertisement content to induce consumers visit their store or click on their ad.

Also note that when search cost b satisfies $b \geq b_3$, then the targeting strategies chosen in Propositions 1 and 2 are identical. Therefore pay-per-click advertising with fixed pay-per-click prices is efficient when search costs b are large enough.

9 Ad Strategy Commitment

Another criticism of my model is that the ad platform publicly reveals its targeting strategy m to the firm and the consumer before the firm chooses its price p and the consumer decides whether or not to click on the ad ω_c. In this section

[30] See Appendix A for a formal proof.

[31] I leave this to future research.

I discuss the credibility of this assumption by analyzing my game (see section 3 with the adaption that the ad platform does not reveal its targeting decision to the firm and the consumers. Instead it announces a targeting strategy m' that may or may not be equal to its targeting strategy m.

Suppose the ad platform announced targeting strategy m' satisfying CM1-4 and the consumers' clicking condition. Further suppose consumers believed the ad platform. Then all consumers shown the ad would click on the ad by the consumers' clicking condition. Knowing this, the ad platform's best response would be to advertise to all consumers, $m = f$, because it wants to maximize the mass $\overline{M}_c$ of consumers clicking on the ad. Therefore the consumers and the firm can only rationally expect the ad platform to advertise to all consumers. Therefore when the ad platform cannot commit to its announced targeting strategy, there is no targeted advertising: the ad platform always shows its ad to all consumers.[32]

10 Concluding Remarks

I found that when the platform is maximizing clicking, it will not show the ad to some consumers that it would rationally expect to buy the product; it will not show the ad to a consumer with a reservation price $r = p \in (p^*, \widehat{p})$ with a probability $1 - f(p^*)(p^* - c)^2/f(p)(p-c)^2$, where p^* is the rationally expected price. And it will show an ad to some consumers that it would rationally expect not to buy the product; it would show the ad to all consumers with reservation prices $r = p \in [\underline{p}, p^*)$. Targeting in this way changes the shape of the demand curve, inducing online merchants to lower their prices p^*. This increases the expected benefit from clicking b_c, leading to more clicking.

Future research in targeted advertising should look at the incentives of online advertising platforms. My model shows that the ad platform could show ads strategically inefficiently and Cornière (2009) shows that an ad platform can over advertise. Neither Cornière (2009) nor my basic model looks at how endogenous advertising prices would affect the ad platform's targeting decision. In Section 8, I showed how take-it-or-leave-it offer pricing can induce the ad platform to target its ads efficiently. Yet in reality, online advertising platforms sell ad space through auctions. Future research should look at how online auctions influence how ad platforms choose to target advertisements.

Also future research needs to test how competition between ad platforms and online merchants affects targeted advertising. Perhaps the kind of strategic inefficiency I found does not exists when multiple online ad platforms (say Yahoo and Google) compete to sell advertisement space to online merchants. Or perhaps the substitution between goods sold by online merchants induces ad platforms to differentiate the ads shown different consumers.

[32] It might be possible to get the ad platform to commit to a targeting strategy m in an infinite period game with a trigger strategy. I leave this to future research.

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A Proofs

Lemma 1. *If $\int_{p^m}^{\infty}(p-p^m)f(p)dp \equiv b_1 \geq b$, then the advertising platform will advertise to every consumer.*

Proof. Because h is continuous and increasing, p^m exists and is unique. If the ad platform were to choose $m = f$ then the consumers' clicking condition becomes $\int_{p^m}^{\infty}(p-p^m)f(p)dp \equiv b_1 \geq b$, which is satisfied by assumption. If the ad platform were to choose $m = f$ then the firm sets its price as p^m by the firm's profit maximizing condition. Therefore if the ad platform were to choose $m = f$ then $\overline{M}_c = 1$, which is its maximum possible value. Therefore it is an equilibrium. It is unique because I do not allow the ad platform to choose any strategies with removable discontinuities. □

Lemma 2. *If $\int_{p^m}^{\infty}(p-p^m)f(p)dp \equiv b_1 < b$ and there exists a targeting strategy $\widetilde{m}$ satisfying CM1-4 that would induce consumers to click, then the advertising platform will not advertise to every consumer.*

Proof. The advertising platform strictly prefers $m = \widetilde{m}$ to $m = f$. Therefore $m = f$ is not optimal. □

Lemma 3. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and either (a), (b) or (c) that produces weakly more clicking.*

a) $m(p) = f(p)\ \forall p \leq p^*$

b) $m(p) = f(p)\ \forall p \in [\underline{p}, p^*]$ *and* $m(p) = 0 \forall p < \underline{p}$

c) $m(p) = 0\ \forall p \leq p^*$

where p^* is the price set by the firm under m and $\underline{p} \in (-\infty, p^*)$.

Proof. Let $\widetilde{p}$ be the price set by the firm under $\widetilde{m}$. Let $\widetilde{M}$ be the cumulative density function of $\widetilde{m}$ and $\overline{\widetilde{M}} \equiv \int_{-\infty}^{\infty}\widetilde{m}(p)dp$. If $\widetilde{m}$ satisfies (a), (b) or (c), then let $m = \widetilde{m}$ and $p^* = \widetilde{p}$. Otherwise $0 < \widetilde{M}(\widetilde{p}) < F(\widetilde{p})$, because $\widetilde{m}$

satisfies CM1 and CM2. If $\widetilde{M}(\widetilde{p}) = F(\widetilde{p})$ then $\widetilde{m}$ would satisfy (c). If $\widetilde{M}(\widetilde{p}) = F(\widetilde{p})$ then $\widetilde{m}$ would satisfy (a). Because F is continuous, choose $\underline{p}$ such that $F(\widetilde{p}) - F(\underline{p}) = \widetilde{M}(\widetilde{p})$. Let $m(p) \equiv f(p) * 1\{\underline{p} \leq p \leq \widetilde{p}\} + m(p) * 1\{p > \widetilde{p}\}$. Because $\overline{M} - M(p) = \overline{\widetilde{M}} - \widetilde{M}(p) \forall p \geq \widetilde{p}$ and the firm weakly prefers setting $\widetilde{p}$ to all prices $p \geq \widetilde{p}$ under the targeting strategy $\widetilde{m}$, the firm will choose its price $p^* \leq \widetilde{p}$ under the targeting strategy m. If the firm would sets price $p^* = \widetilde{p}$ under the targeting strategy m, then m satisfies (b). If the firm would set its price $p^* < \widetilde{p}$ under the targeting strategy m, then m adds more slackness to the consumers' clicking condition because:

$$\int_{p^*}^{\infty} (p-p^*)m(p)dp = \int_{\widetilde{p}}^{\infty} (p-\widetilde{p})\widetilde{m}(p)dp + (\widetilde{p}-p^*)(\overline{\widetilde{M}} - \widetilde{M}(\widetilde{p})) + \int_{p^*}^{\widetilde{p}} (p-p^*)f(p)dp$$

$$> \int_{\widetilde{p}}^{\infty} (p-\widetilde{p})\widetilde{m}(p)dp \tag{6}$$

Therefore if consumers would click on the ad under the targeting strategy $\widetilde{m}$, then consumers would click on the ad under targeting strategy m. $\square$

Lemma 4. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and $m(p^*) = f(p^*)$ that produces weakly more clicking, where p^* is the price set by the firm under m.*

Proof. Let $\widetilde{p}$ be the price set by the firm under $\widetilde{m}$. Let $\widetilde{M}$ be the cumulative density function of $\widetilde{m}$ and $\overline{\widetilde{M}} \equiv \int_{-\infty}^{\infty} \widetilde{m}(p)dp$. If $\widetilde{m}(\widetilde{p}) = f(\widetilde{p})$, then let $m = \widetilde{m}$ and $p^* = \widetilde{p}$. If $\widetilde{M}(\widetilde{p}) > 0$, then such an m exists by Lemma 3. If $\widetilde{m}(\widetilde{p}) \neq f(\widetilde{p})$ and if $\widetilde{M}(\widetilde{p}) = 0$, then let $m_\epsilon(p) \equiv \widetilde{m}(p) * 1\{p > \widetilde{p}\} + f(p) * 1\{\widetilde{p} - \epsilon \leq p \leq \widetilde{p}\}$ $\forall \epsilon > 0$. Let p_ϵ be the price set by the firm under m_ϵ. Let M_ϵ be the cumulative density function of m_ϵ and $\overline{M_\epsilon} \equiv \int_{-\infty}^{\infty} m_\epsilon(p)dp$. By the firm's profit maximizing condition $(\widetilde{p} - c)\widetilde{m}(\widetilde{p}) \geq \overline{\widetilde{M}} - \widetilde{M}(\widetilde{p})$. Therefore $(\widetilde{p} - c)m_\epsilon(\widetilde{p}) = (\widetilde{p} - c)f(\widetilde{p}) > \overline{\widetilde{M}} - \widetilde{M}(\widetilde{p}) = \overline{M}_\epsilon - M_\epsilon(\widetilde{p})$ $\forall \epsilon > 0$. Hence $p_\epsilon < \widetilde{p}$ by the firm's profit maximizing condition. Choose an arbitrary small $\epsilon > 0$ such that $(\widetilde{p} - \epsilon - c)f(\widetilde{p} - \epsilon) > \overline{\widetilde{M}} - \widetilde{M}(\widetilde{p}) + F(\widetilde{p}) - F(\widetilde{p} - \epsilon) = \overline{M}_\epsilon - M_\epsilon(\widetilde{p} - \epsilon)$. Therefore $p_\epsilon = \widetilde{p} - \epsilon$. Let $m = m_\epsilon$ and $p^* = p_\epsilon$. Note that m adds more slackness to the consumers' clicking condition because of equation (6). Therefore if consumers would click on the ad under targeting strategy with the density function $\widetilde{m}$, then consumers would click on the ad under targeting strategy with the density function m. $\square$

Lemma 5. *Given any prices p' and p'' satisfying $p'' < p'$ it follows that $f(p') > f(p'')(p'' - c)^2/(p' - c)^2$.*

Proof. Let $K \equiv f(p'')(p'' - c)^2$. $f(p') > K/(p' - c)^2$, because $1 - F$ is log-concave, any cumulative function of a density function of the form $K/(p' - c)^2$ is log-convex and $f(p'') = K/(p'' - c)^2$. $\square$

Lemma 6. *Given any targeting strategy m satisfying CM1-4, it follows that $(p^* - c)(M(p) - M(p^*))/(p - p^*) \geq \overline{M} - M(p)$ $\forall p > p^*$, where p^* is the rationally expected price under m, M is the cummulative funtion of m and $\overline{M} \equiv \int_{-\infty}^{\infty} m(p)dp$.*

Proof. By the firm's first order condition $(p^* - c)(\overline{M} - M(p^*)) \geq (p - c)(\overline{M} - M(p))$. □

Lemma 7. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and either (a), (b) or (c) that produces weakly more clicking.*

a) $m(p) = f(p)$ $\forall p > p^*$

b) $m(p) = K/(p - c)^2$ $\forall p \in (p^*, \widehat{p}]$ *and* $m(p) = f(p)$ $\forall p > \widehat{p}$

c) $m(p) = K/(p - c)^2$ $\forall p > p^*$

where p^* is the price set by the firm under m, $K \equiv f(p^*)(p^* - c)^2$ and $\widehat{p} \in (p^*, \infty)$.

Proof. Let $\widetilde{p}$ be the price set by the firm under $\widetilde{m}$. Let $\widetilde{M}$ be the cumulative density function of $\widetilde{m}$ and $\overline{\widetilde{\widetilde{M}}} \equiv \int_{-\infty}^{\infty} \widetilde{m}(p)dp$. If $\widetilde{m}$ satisfies (a), (b) or (c), then let $m = \widetilde{m}$ and $p^* = \widetilde{p}$. If no consumers would be induced to click under $\widetilde{m}$, then any targeting strategy is weakly prefereable to $\widetilde{m}$, so choose m that satisfies CM1-4 and (a), (b) or (c). Otherwise $\overline{\widetilde{\widetilde{M}}} - \widetilde{M}(p^*) < 1 - F(p^*)$ and consumers are induced to click under $\widetilde{m}$. By Lemma 4 choose a targeting strategy m_1 that produces weakly more clicking than $\widetilde{m}$ satisfying CM1-4 and $m(p^*) = f(p^*)$, where p^* is the rationally expected price under m_1. Let M_1 be the cumulative density function of m_1 and $\overline{M} \equiv \int_{-\infty}^{\infty} m_1(p)dp$. Let $K \equiv f(p^*)(p^* - c)^2$, $g(p) \equiv K/(p - c)^2$ and $G(p) \equiv \overline{M} - K/(p - c)$. Note that by Lemma 5, $g(p) \leq f(p)$ $\forall p \geq p_1$.

Because m_1 has no removable, infinite or essential discontinuities and only finite jump discontinuities, I split (p^*, ∞) into a countable series of intervals such that: 1) m_1 is either entirely weakly above or entirely weakly below the curve g_1 for any given interval, 2) no two weakly above intervals border eachother (I would join these two intervals together to make one interval), and 3) no two weakly below intervals border eachother. Let $\{p_k\}_{k=0,1,...}$ be the sequence of prices at the bounds of these intervals. Note $p_0 = p^*$.

The first such interval $(p^*, p_1]$ is necessarily weakly above g_1. Otherwise $M_1(p_1) - M_1(p^*) < G(p_1) - G(p^*)$. Because g solves the differential equation for $(p^* - c)(G(p) - G(p^*))/(p - p^*) < \overline{M} - G(p)$, I would have $(p^* - c)(M_1(p) - M_1(p^*))/(p - p^*) < \overline{M} - M_1(p)$ which I have shown would violate the profit maximizing condition for the firm in Lemma 6. Also by the same reasoning, $M_1(p_2) - M_1(p_1) \geq G(p_2) - G(p_1)$. That is the area above g in the first interval has to be bigger than the area above g in the second interval. Therefore I choose $\widetilde{p}_1$ satisfying $M_1(p_2) - M_1(p^*) = G(p_1) - G(p^*) + F(p_2) - F(p_1)$.

Define $m_2(p) \equiv g(p) * 1\{p^* < p \leq \widetilde{p}_1\} + f(p) * 1\{\widetilde{p}_1 < p \leq p_2\} + m(p) * 1\{p \notin (p^*, p_2]\}$. Note that the firm would set price p^* under m_2. For prices outside the interval $(p^*, p_2]$, the firm's demand curve would not change. For prices inside the interval $(p^*, p_2]$, the firm would have less incentive for raising its price above p^*. Also note that the consumers before clicking would be weakly better off, because the expected value of clicking, b_c, would increase while the mass of consumers buying, Q, would remain constant.

Now m_2 on the interval $(p^*, p_3]$ is necessarily weakly above g, m_2 on the interval $(p_3, p_4]$ is necessarily weakly below g, and so on. I repeat the same process I used to define m_2 from m_1 to define m_3 from m_2. I iterate over this to get a function of form (b). If this takes finite iterations then my terminating function is of the form (b). I define this as the targeting strategy m. If this takes infinite iterations then I use the limit of the subsequence of the functions (which must exist because my choice of a function is bounded). Let me call this function $\widehat{m}$. $\widehat{m}$ would not have any essential or infinite discontinuities because $\widehat{m}$ is bounded and the slope of $\widehat{m}$ is bounded from below. I eliminate all the removable discontinuities from $\widehat{m}$ and revalue my jump discontinuities by CM3 and CM4. I call this function m, which satisfies (b). □

Lemma 8. *Given any targeting strategy $\widetilde{m}$ satisfying CM1-4, there exists a targeting strategy m satisfying CM1-4 and either (a), (b), (c) or (d) that produces weakly more clicking.*

a) $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$

b) $m(p) = f(p) * 1\{p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$

c) $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p-c)^2 * 1\{p > p^*\}$

d) $m(p) = f(p) * 1\{p \leq p^*\} + K/(p-c)^2 * 1\{p > p^*\}$

where p^* is the rationally expefect prices under m, $K \equiv f(p^*)(p^* - c)^2$, $\widehat{p} \in [p^*, \infty)$ and $\underline{p} \in (-\infty, p^*]$.

Proof. This follows directly from Lemmas 3, 4 and 7. □

Lemma 9. *If $\int_{p^m}^{\infty}(p - p^m)f(p)dp \equiv b_1 < b$ and there exists a targeting strategy $\widetilde{m}$ satisfying CM1-4 that would induce consumers to click, then the advertising platform would choose a targeting strategy such that $b_c = b$.*

Proof. By way of contradiction suppose not. Suppose $b > b_1$ and there exists a targeting strategy $\widetilde{m}$ that would induce consumers to click satisfying CM1-4. Suppose there exists a click maximizing targeting strategy m satisfying CM1-4 and $b_c = \int_{p^*}^{\infty}(p - p^*)m(p)dp \neq b$ where p^* be the price set by the firm under m. Let M be the cumulative density function of m and $\overline{M} \equiv \int_{-\infty}^{\infty} m(p)dp$. By the consumers' clicking condition if $b_c < b$ then m would not induce any consumers to click on the ad, so $b_c > b$. By Lemma 2, the advertisement platform shows the ad to some but not all of the consumers so $0 < \overline{M} < 1$. Because $\overline{M} < 1$ and

m has only finite discontinuities, choose a price p' such that $m(p') < f(p')$ and m is continuous at p'. Define $m_\epsilon(p) \equiv m(p) * 1\{p \leq p - \epsilon\} + f(p) * 1\{p - \epsilon < p \leq p + \epsilon\} + m(p) * 1\{p > p + \epsilon\}$ $\forall \epsilon > 0$. Let p_ϵ be the price set by the firm under m_ϵ. Let M_ϵ be the cumulative density function of m_ϵ and $\overline{M}_\epsilon \equiv \int_{-\infty}^{\infty} m_\epsilon(p)dp$ $\forall \epsilon$. Because m is continuous at p', choose an $\epsilon > 0$ such that $\int_{p_\epsilon}^{\infty}(p - p_\epsilon)m_\epsilon(p)dp > b$. Consumers would still be induced to click under m_ϵ by the consumer's clicking condition and $\overline{M}_\epsilon > \overline{M}$. Thus the advertisement platform strictly prefers the feasible targeting strategy m_ϵ to m. □

Proposition 1. *There exists a click-maximizing targeting strategy m satisfying CM1-4 and the following:*

a) when $b \leq b_1$: $m = f$;

b) when $b_1 < b \leq b_2$: $m(p) = f(p) * 1\{p \leq p^*\} + K/(p - c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ where $\widehat{p} \in (p^m, p_1]$;

c) when $b_2 < b \leq b_3$: $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p - c)^2 * 1\{p^* < p \leq p_1\} + f(p) * 1\{p > p_1\}$ where $\underline{p} \leq p^*$;

d) when $b_3 < b \leq b_4$: $m(p) = K/(p - c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ where $\widehat{p} \in (p_1, p_2]$;

e) when $b > b_4$: no market exists so any m is optimal;

for some $0 < b_1 < b_2 < b_3 < b_4$ where b is the search cost, p^* is the rationally expected price, $p_2 > p_1 > p^*$ and $K \equiv f(p^*)(p^* - c)^2$.

Proof. by parts:

Part 1. *When $b \leq b_1$: $m = f$.*

Define $b_1 \equiv \int_{p^m}^{\infty}(p - p^m)f(p)dp$, where p^m is the price set by the firm facing the demand curve $1 - F$. By Lemma 1, if $bb1$, then the unique optimal targeting strategy is $m = f$.

Part 2. *The search costs $b > b_1$ such that there exists a targeting strategy $\widetilde{m}$ satisfying CM1-4 that would induce consumers to click form a contiguous interval with an infimum of b_1.*

Define $b_s \equiv b_1/(1 - F(p^m))$, where p^m is the price set by the firm facing the demand curve $1 - F$. By the consumers clicking condition, if the search cost b were less than or equal to b_s, then the consumers would click under the targeting strategy $m(p) = f(p) * 1\{p \geq p^m\}$. Because $b_s > b_1$, there exists search costs $b > b_1$ for which the targeting strategy $m = f$ does not satisfy the consumers' clicking condition and at least one other targeting strategy $\widetilde{m}$ satisfies the consumers' clicking condition.

If the ad platform has a strategy that can induce some consumers to click for search cost b', then it can use the same strategy to induce some consumers to click for search cost $b'' < b'$. Therefore, by Lemma 2, the search costs $b > b_1$ such that there exists a targeting strategy $\widetilde{m}$ satisfying CM1-4 that would induce consumers to click form a contiguous interval with an infimum of b_1.

Part 3. *When $\underline{p} < p^*(\underline{p}, \widehat{p})$, given any pair of $\underline{p}$ and $\widehat{p} \in [p^*, p_1)$ there exists another pair $\underline{p}'$ and $\widehat{p}' \in [p^*, p_1)$ satisfying $\overline{M}(\underline{p}, \widehat{p}) = \overline{M}(\underline{p}', \widehat{p}')$ and $b_c(\underline{p}', \widehat{p}') > b_c(\underline{p}, \widehat{p})$.*

Define $m[\underline{p}, \widehat{p}](p) \equiv f(p) * 1\{\underline{p} \leq p \leq p^*(\underline{p}, \widehat{p})\} + K(p^*(\underline{p}, \widehat{p}))/(p-c)^2 * 1\{p^*(\underline{p}, \widehat{p}) < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$, where $p^*(\underline{p}, \widehat{p})$ is the rationally expected price under $m[\underline{p}, \widehat{p}]$ and $K(p) \equiv f(p)(p-c)^2 \ \forall p \in \Re$. Let $M[\underline{p}, \widehat{p}]$ be the cumulative density function of $m[\underline{p}, \widehat{p}]$ and $\overline{M}[\underline{p}, \widehat{p}] \equiv \int_{-\infty}^{\infty} m[\underline{p}, \widehat{p}](p)dp$. Because f is continuous, $m[\underline{p}, \widehat{p}]$ satisfies CM1-4 for any $\underline{p}$ and $\widehat{p}$.

I will begin by analyzing the case where $\underline{p} < p^*(\underline{p}, \widehat{p})$. In this case changing $\underline{p}$ does not change $p^*(\underline{p}, \widehat{p})$, because the firm would not choose to sell to those consumers anyway (see the firm's profit maximizing condition), so I write $p^*(\widehat{p})$.

The advertisement platform can increase the expected benefit b_c of clicking on the ad in two ways: through increasing $\underline{p}$ and through increasing $\widehat{p}$. Doing so decreases the mass $\overline{M}[\underline{p}, \widehat{p}]$ of consumers clicking. The decrease in the mass from increasing one choice variable can be offset by the increase in the mass from decreasing the other choice variable. The effect of changing the expected benefit b_c of clicking through increasing $\underline{p}$ (and holding $\widehat{p}$ constant) is given in equation (7) and through increasing $\widehat{p}$ (and holding $\underline{p}$ constant) is given in equation (8).

$$\frac{\partial b_c}{\partial \overline{M}[\underline{p}, \widehat{p}]}(\underline{p}, \widehat{p})\bigg|_{through\ \underline{p}} = \frac{\partial b_c}{\partial \underline{p}}(\underline{p}, \widehat{p}) \bigg/ \frac{\partial \overline{M}[\underline{p}, \widehat{p}]}{\partial \underline{p}}(\underline{p}, \widehat{p}) = -X(\underline{p}, \widehat{p}) \tag{7}$$

$$\text{where } X(\underline{p}, \widehat{p}) \equiv \textstyle\int_{p^*(\widehat{p})}^{\infty} (p - p^*(\widehat{p}))m[\underline{p}, \widehat{p}](p)dp \big/ \overline{M}[\underline{p}, \widehat{p}]^2$$

$$\frac{\partial b_c}{\partial \overline{M}[\underline{p}, \widehat{p}]}(\underline{p}, \widehat{p})\bigg|_{through\ \widehat{p}} = \frac{\partial b_c}{\partial \widehat{p}}(\underline{p}, \widehat{p}) \bigg/ \frac{\partial \overline{M}[\underline{p}, \widehat{p}]}{\partial \widehat{p}}(\underline{p}, \widehat{p}) = \frac{\partial CS[\underline{p}, \widehat{p}]/\partial \widehat{p}}{\overline{M}[\underline{p}, \widehat{p}] * \partial \overline{M}[\underline{p}, \widehat{p}]/\partial \widehat{p}} - X(\underline{p}, \widehat{p}) \tag{8}$$

$$\text{where } CS[\underline{p}, \widehat{p}] \equiv \textstyle\int_{p^*(\widehat{p})}^{\infty} (p - p^*(\widehat{p}))m[\underline{p}, \widehat{p}](p)dp$$

$$\text{Note: } b_c = CS[\underline{p}, \widehat{p}]/\overline{M}[\underline{p}, \widehat{p}]$$

Here, $X(\underline{p}, \widehat{p})$ is the increase in b_c through decreasing the mass $\overline{M}[\underline{p}, \widehat{p}]$ of consumers clicking by increasing $\underline{p}$. Because $p^*(\widehat{p})$ and $m[\underline{p}, \widehat{p}](p)$ above $p^*(\widehat{p})$ does not depend on $\underline{p}$, changing $\underline{p}$ does not affect the Consumer Surplus $CS[\underline{p}, \widehat{p}]$, so changing $\underline{p}$ only affects b_c through decreasing the mass $\overline{M}[\underline{p}, \widehat{p}]$. This affect is $X(\underline{p}, \widehat{p})$.

Yet, changing $\widehat{p}$ changes p^*, so changing $\widehat{p}$ changes b_c through changing $\overline{M}[\underline{p}, \widehat{p}]$ and $CS[\underline{p}, \widehat{p}]$. Equation (9) decomposes $\partial CS[\underline{p}, \widehat{p}]/\partial \widehat{p}$.

$$\frac{\partial CS[\underline{p}, \widehat{p}]}{\partial \widehat{p}} = PE(\widehat{p}) - Y(\widehat{p}) - Z(\widehat{p}) \tag{9}$$

$$\text{where } PE(\widehat{p}) \equiv -(1-F(\widehat{p}))\tfrac{\partial p^*(\widehat{p})}{\partial \widehat{p}}(\widehat{p}) > 0$$

$$\text{where } Y(\widehat{p}) = -\textstyle\int_{p^*(\widehat{p})}^{\widehat{p}} \tfrac{\partial}{\partial \widehat{p}}[(p-p^*(\widehat{p}))\tfrac{K(p^*(\widehat{p}))}{(p-c)^2}]dp$$

$$\text{where } Z(\widehat{p}) = (p-p^*(\widehat{p}))(f(\widehat{p}) - \tfrac{K(p^*(\widehat{p}))}{(p-c)^2}) > 0$$

Here $PE(\widehat{p})$ is the *price effect* on all consumers with reservation prices above $\widehat{p}$. It reflects how much each consumer with a reservation price above $\widehat{p}$ will benefit from the lowering of the price set by the firm. $Y(\widehat{p})$ is the *infra-marginal consumer loss effect*. By increasing $\widehat{p}$, the ad platform is inducing the firm to choose a lower price $p^*(\widehat{p})$. By lowering $p^*(\widehat{p})$, the constant $K(p^*(\widehat{p}))$ is lower. This in turn lowers the ad platforms choice of $m[\underline{p},\widehat{p}]$ between $p^*(\widehat{p})$ and $\widehat{p}$. $Z(\widehat{p})$ is the *marginal consumer loss effect*. By increasing $\widehat{p}$, the advertisement platform is not advertising to some consumers with reservation prices $r = \widehat{p}$. $Z(\widehat{p})$ captures the effect of the loss of the advertising to these consumers on the consumer surplus.

Note that this implies that $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ is independant of $\underline{p}$.

As the difference $\widehat{p} - p^*(\widehat{p})$ approaches zero: the infra-marginal consumer loss effect $Y(\widehat{p})$ and the marginal consumer loss effect $Z(\widehat{p})$ converge to zero. Thus as $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ becomes the price effect $PE(\widehat{p}) > 0$. Thus $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ is strictly greater than zero.

As the difference $\widehat{p} - p^*(\widehat{p})$ approaches infinity: the price effect $PE(\widehat{p})$ converges to zero, the infra-marginal consumer loss effect $Y(\widehat{p})$ remains bounded and the marginal consumer loss effect $Z(\widehat{p})$ goes to infinity. Thus $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ goes to negative infinity.

Because $PE(\widehat{p})$, $Y(\widehat{p})$ and $Z(\widehat{p})$ are continuous with respect to changes in $\widehat{p}$, $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ is continuous with respect to changes in $\widehat{p}$. Therefore by the Intermediate Value Theorem, there exists at least one p_1 such that $\partial CS[\underline{p},p_1]/\partial\widehat{p} = 0$. For any p_1 that satisfies this, I have $\widehat{p} - p^*(\widehat{p} = p_1) > 0$, because $\partial CS[\underline{p},0]/\partial\widehat{p} > 0$.

If there is more than one possible candidate p_1, then define p_1 as the one that maximizes $CS[\underline{p},\widehat{p}]$. In the case of a tie, pick the first.

Therefore when $\underline{p} < p^*(\underline{p},\widehat{p})$, the ad platform can increase b_c, while holding the mass $\overline{M}$ of consumers clicking constand by choosing a smaller $\underline{p}$ and a larger $\widehat{p}$ to compensate for the $\overline{M}$ gained due to a smaller $\underline{p}$.

Part 4. *When $\underline{p} = p^*(\underline{p},\widehat{p})$, given any pair of $\underline{p}$ and $\widehat{p} \in [p^*, p_1)$ there exists another pair $\underline{p}'$ and $\widehat{p}' \in [p^*, p_1)$ satisfying $\overline{M}(\underline{p},\widehat{p}) = \overline{M}(\underline{p}',\widehat{p}')$ and $b_c(\underline{p}',\widehat{p}') > b_c(\underline{p},\widehat{p})$.*

Define $m[\underline{p},\widehat{p}]$, $p^*(\underline{p},\widehat{p})$, $K(p)$, $M[\underline{p},\widehat{p}]$ and $\overline{M}[\underline{p},\widehat{p}]$ as in Part 3.

This uses the argument in Part 3, with a caveat: I need to show that the ad platform would never set $\underline{p}$ above the limit price $\lim_{\underline{p}\to p^*} p^*(\underline{p},\widehat{p})$. Obviously if this property is true for all $\underline{p} < p^*(\underline{p},\widehat{p})$, then it is true for the limit case.

If $\underline{p} = p^*(\underline{p},\widehat{p})$ and we are not in the limit case then $(\underline{p} - c)m[\underline{p},\widehat{p}](\underline{p}) > \overline{M}[\underline{p},\widehat{p}] - M[\underline{p},\widehat{p}](\underline{p})$, by equation (3). Therefore choose an $\epsilon > 0$ small enough so $\underline{p} - \epsilon = p^*(\underline{p} - \epsilon,\widehat{p})$ and $(\underline{p} - \epsilon - c)m[\underline{p} - \epsilon,\widehat{p}](\underline{p} - \epsilon) > \overline{M}[\underline{p} - \epsilon,\widehat{p}] - M[\underline{p} -$

$\epsilon, \widehat{p}](\underline{p}-\epsilon)$. Targeting strategy $m[\underline{p}-\epsilon, \widehat{p}]$ would produce a higher b_c than $m[\underline{p}, \widehat{p}]$ and $\overline{M}[\underline{p}-\epsilon, \widehat{p}] > \overline{M}[\underline{p}, \widehat{p}]$.

Therefore when $\underline{p} = p^*(\underline{p}, \widehat{p})$, the ad platform can increase b_c, while holding the mass $\overline{M}$ of consumers clicking constand by choosing a smaller $\underline{p}$ and a larger $\widehat{p}$ to compensate for the $\overline{M}$ gained due to a smaller $\underline{p}$.

Part 5. *When* $b_1 < b \leq b_2$*:* $m(p) = f(p) * 1\{p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ *where* $\widehat{p} \in (p^m, p_1]$

Define $m[\underline{p}, \widehat{p}]$, $p^*(\widehat{p})$, $K(p)$, $M[\underline{p}, \widehat{p}]$ and $\overline{M}[\underline{p}, \widehat{p}]$ as in Part 3.

Define $b_2 \equiv \Big(\int_{p^*(p_1)}^{p_1} (p - p^*(p_1)) \frac{K(p^*(p_1))}{(p-c)^2} dp + \int_{p_1}^{\infty} (p - p^*(p_1)) f(p) dp \Big) / \Big(1 - F(p_1) + \frac{K(p^*(p_1))}{p^*(p_1)-c} + \frac{K(p^*(p_1))}{p_1-c} + F(p^*(p_1)) \Big)$

By the consumers' clicking condition, if the search cost $b \leq b_2$, then the consumers would click under the targeting strategy $m_2(p) \equiv f(p) * 1\{p \leq p^*(p_1)\} + K(p^*(p_1)/(p-c)^2 * 1\{p^*(p_1) < p \leq p_1\} + f(p) * 1\{p > p_1\}$.

By Parts 3 and 4, if the search cost b satisfies $b_1 < b_2 < b2$, then the click maximizing targeting strategy that gives just enough expected benefit b_c to get consumers to click would be for $\underline{p}$ to be as low as possible; in this case, $\underline{p} \to \infty$.

Part 6. *When* $\underline{p} < p^*(\underline{p}, \widehat{p})$*, given any pair of* $\underline{p}$ *and* $\widehat{p} \in (p_1, p_2]$ *there exists another pair* $\underline{p}'$ *and* $\widehat{p}' \in (p_1, p_2]$ *satisfying* $\overline{M}(\underline{p}, \widehat{p}) = \overline{M}(\underline{p}', \widehat{p}')$ *and* $b_c(\underline{p}', \widehat{p}') > b_c(\underline{p}, \widehat{p})$*.*

This follows directly from the argument given in Part 3. For $\widehat{p} > p_1$, $\partial CS[\underline{p}, \widehat{p}]/\partial \widehat{p} < 0$.

Part 7. *When* $b_2 < b \leq b_3$*:* $m(p) = f(p) * 1\{\underline{p} \leq p \leq p^*\} + K/(p-c)^2 * 1\{p^* < p \leq p_1\} + f(p) * 1\{p > p_1\}$ *where* $\underline{p} \leq p^*$

Define $m[\underline{p}, \widehat{p}]$, $p^*(\widehat{p})$, $K(p)$, $M[\underline{p}, \widehat{p}]$ and $\overline{M}[\underline{p}, \widehat{p}]$ as in Part 3.

Define $b_3 \equiv \Big(\int_{p^*(p_1)}^{p_1} (p - p^*(p_1)) \frac{K(p^*(p_1))}{(p-c)^2} dp + \int_{p_1}^{\infty} (p - p^*(p_1)) f(p) dp \Big) / \Big(1 - F(p_1) + \frac{K(p^*(p_1))}{p^*(p_1)-c} + \frac{K(p^*(p_1))}{p_1-c} \Big)$.

By the consumers' clicking condition, if the search cost $b \leq b_3$, then the consumers would click under the targeting strategy $m_3(p) \equiv K(p^*(p_1)/(p-c)^2 * 1\{p^*(p_1) < p \leq p_1\} + f(p) * 1\{p > p_1\}$.

By Part 6, if the search cost b satisfies $b_2 \leq b \leq b_3$, the click maximizing targeting strategy would have $\widehat{p} = p_1$. The price cutoff $\underline{p}$ would be just low enough to give the expected benefit b_c equal to the search cost b.

Part 8. *When* $b_3 < b \leq b_4$*:* $m(p) = K/(p-c)^2 * 1\{p^* < p \leq \widehat{p}\} + f(p) * 1\{p > \widehat{p}\}$ *where* $\widehat{p} \in (p_1, p_2]$.

After increasing $\widehat{p}$ to p_1 and increasing $\underline{p}$ to p^*, then the expected benefit of clicking, b_c , still might be large enough to induce the consumer to click on the ad. As long as the equation (8) is negative, the ad platform can still increase the expected benefit from clicking on the ad by increasing $\widehat{p}$. Increasing $\widehat{p}$ further still increases the expected benefit from clicking on the ad through decreasing the mass of consumers clicking on the ad or equivalently through $X(\underline{p}, \widehat{p})$. Yet it is opposed by $\partial CS[\underline{p}, \widehat{p}]/\partial \widehat{p}$. When $\big|\partial CS[\underline{p}, \widehat{p}]/\partial \widehat{p}\big| > \big|X(\underline{p}, \widehat{p}) * \overline{M}[\underline{p}, \widehat{p}] * \partial \overline{M}[\underline{p}, \widehat{p}]/\partial \widehat{p}\big|$, then the advertisement platform can still increase the expected benefit of clicking on the ad by increasing $\widehat{p}$.

I have established that $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ is zero when $\widehat{p}=p_1$ and negative infinity as $\widehat{p}\to\infty$. Because of the continuity of $\partial CS[\underline{p},\widehat{p}]/\partial\widehat{p}$ and $X(\underline{p},\widehat{p})*\overline{M}[\underline{p},\widehat{p}]*\partial\overline{M}[\underline{p},\widehat{p}]/\partial\widehat{p}$, there must exist a $p_2>p_1$ such that $\partial CS[\underline{p},p_2]/\partial\widehat{p}=X(\underline{p},p_2)*\overline{M}[\underline{p},p_2]*\partial\overline{M}[\underline{p},p_2]/\partial\widehat{p}$. If there is more than one local maximum candidate p_2, then at least one would be a global maximum that maximizes the expected benefit from clicking on the ad. The advertising platform would choose the lowest such global maximum which I will refer to herein as p_2.

Define $b_4\equiv\Big(\int_{p^*(p_2)}^{p_2}(p-p^*(p_2))\frac{K(p^*(p_2))}{(p-c)^2}dp+\int_{p_2}^{\infty}(p-p^*(p_2))f(p)dp\Big)/\Big(1-F(p_2)+\frac{K(p^*(p_2))}{p^*(p_2)-c}+\frac{K(p^*(p_2))}{p_2-c}\Big)$.

By the consumers' clicking condition, if the search cost $b\leq b_4$, then the consumers would click under the targeting strategy $m_4(p)\equiv K(p^*(p_2)/(p-c)^2*1\{p^*(p_2)<p\leq p_2\}+f(p)*1\{p>p_2\}$.

By the argument in Part 3, if the search cost b satisfies $b_3\leq b\leq b_4$, the click maximizing targeting strategy would want to lower $\widehat{p}$ to increase the expected benefit b_c to b.

Part 9. *When $b>b_4$: no market exists so any m is optimal.*

If $b>b_4$ then it is impossible for the advertising platform to induce the consumers to click on the ad. If there were a strategy m'' then there would be a strategy of the form given in Lemma 8 that could induce consumers to click on the ad. But my comparative static results in Part 2 rule out this possibility. ☐

Lemma 10. *When $b\geq\int_{p^m}^{\infty}(p-p^m)f(p)dp/(1-F(p^m))$, there exists a firm-profit-maximizing targeting strategy m satisfying CM1-4 and $m(p)=f(p)*1\{p\geq p^m\}$.*

Proof. Suppose not. Let $\widetilde{m}$ satisfying CM1-4 maximize the firm's profits. Let $\widetilde{p}$ be the price set by the firm under $\widetilde{m}$. Let $\widetilde{M}$ be the cumulative density function of $\widetilde{m}$ and $\widetilde{\overline{M}}\equiv\int_{-\infty}^{\infty}\widetilde{m}(p)dp$.

Under the demand function $1-F$, the firm would choose to set its price as p^m. Therefore by the Weak Axiom of Profit Maximization: $(p^m-c)(1-F(p^m))\geq(\widetilde{p}-c)(1-F(\widetilde{p}))$. I have that $1-F(\widetilde{p})\geq\widetilde{\overline{M}}-\widetilde{M}(\widetilde{p})$, by CM1. Thus $(p^m-c)(1-F(p^m))\geq(\widetilde{p}-c)(\widetilde{\overline{M}}-\widetilde{M}(\widetilde{p}))$

Note that consumers would click under the targeting strategy $m_0(p)\equiv f(p)*1\{p\geq p^m\}$ by the consumer's clicking condition. And that this gives the firm the profit $(p^m-c)(1-F(p^m))$. ☐

Proposition 2. *There exists a firm-profit-maximizing targeting strategy m satisfying CM1-4 and the following:*

a) when $b\leq b_5$: $m(p)=f(p)*1\{p\geq p^m\}$;

b) when $b_5\leq b\leq b_4$: $m(p)=K/(p-c)^2*1\{p^*<p\leq\widehat{p}\}+f(p)*1\{p>\widehat{p}\}$ where $\widehat{p}\in(p^m,p_2]$;

c) when $b>b_4$: no market exists so any m is optimal;

for some $0 < b_5 < b_4$ *where* b *is the search cost,* p^* *is the rationally expected price,* $p_2 > p^m$ *and* $K \equiv f(p^*)(p^* - c)^2$.

Proof. Define $b_5 \equiv \int_{p^m}^{\infty}(p - p^m)f(p)dp/(1 - F(p^m))$, where p^m is the price set by the firm facing the demand curve $1 - F$. And define b_4 as in Part 8 of the proof of Proposition 1.

When $b \leq b_5$, consumers would click under the targeting strategy $m_0(p) \equiv f(p) * 1\{p \geq p^m\}$ by the consumer's clicking condition. This maximizes the firms profit by Lemma 10.

When $b_5 \leq b \leq b_4$, consumers would not click under m_0, so the ad platform has to choose a targeting strategy that commits the firm to a lower price to encourage them to click. By Lemma 9, the ad platform would do so to make $b_c = b$. By Lemma 8, the ad platform would do so with a targeting strategy of the form (a), (b), (c) or (d) of Lemma 8. Raising $\underline{p}$ would not affect the firm's profit as long as $\underline{p} < p^*$, so the ad platform would set $\underline{p}$ equal to the rationally expected price p^* of the firm. But doing so would not be enough (because m_0 does not induce consumers to click), so the ad platform would raise $\widehat{p}$ above p^m. Because $b \leq b_4$, it is possible to induce consumers to click with a high enough $\widehat{p}$.

When $b > b_4$, by the argument in Part 9 of the proof of Proposition 1, it is impossible for the ad platform to induce consumers to click. $\square$